

# Understanding the world collectively

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## **‘Proofs’ that there is collective intelligence in general**

Condorcet jury model

Galton experiment

Collective error is always smaller

I will show they are NOT proofs, but each will give us a lesson of what is needed for collective intelligence

**How do animals do it? A couple of lessons from animals**

**Can we enhance it with AI?**

# Condorcet (1785)



**Marquis de Condorcet**

*Essay on the applicability of  
probability analysis to majority decisions*



# Condorcet (1785)

Incorrect option



Correct option



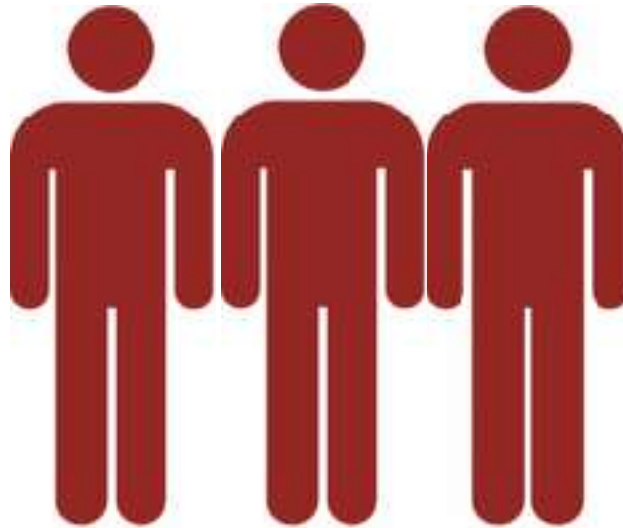
She chooses the right option  
with probability  $p$

# Condorcet (1785)

Incorrect option

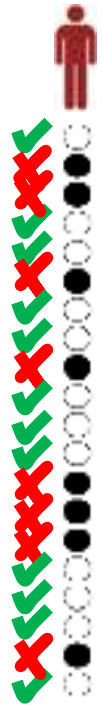


Correct option

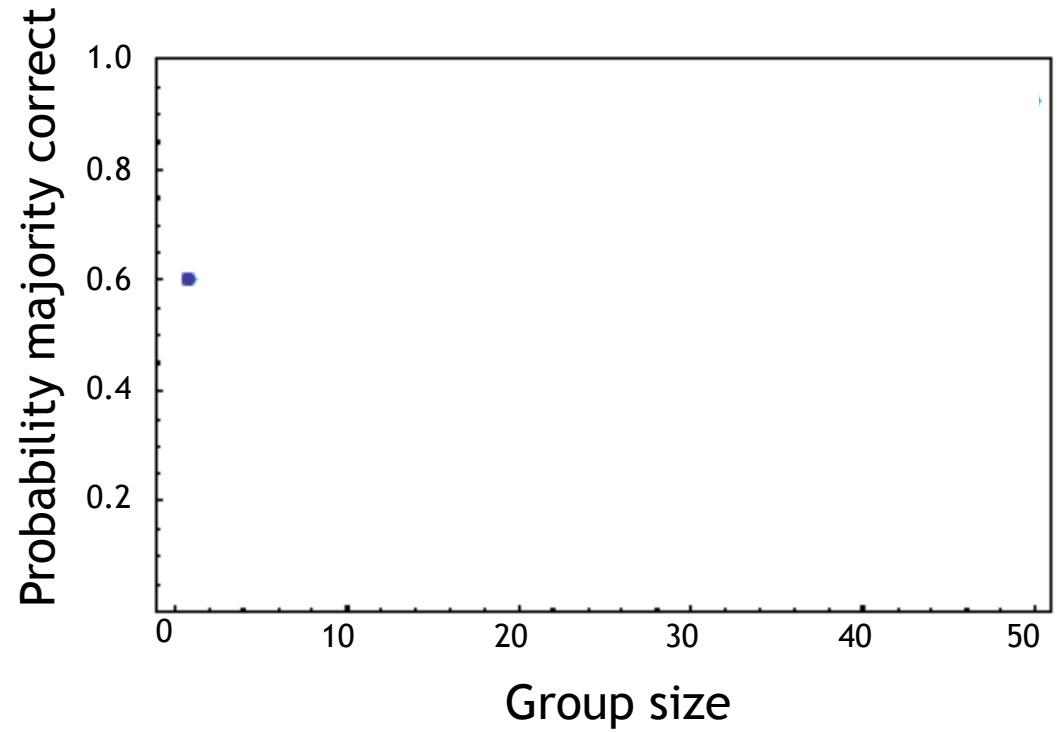


What is the probability that the majority (2 of them) chooses the correct option if each person does with probability  $p$  ?

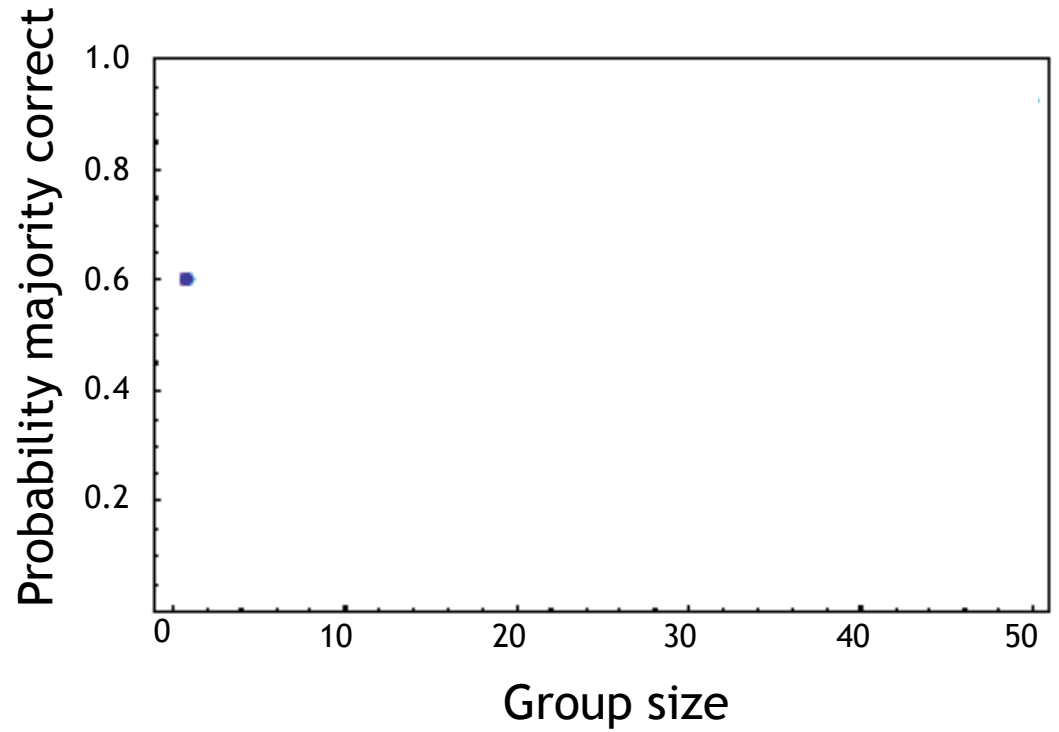
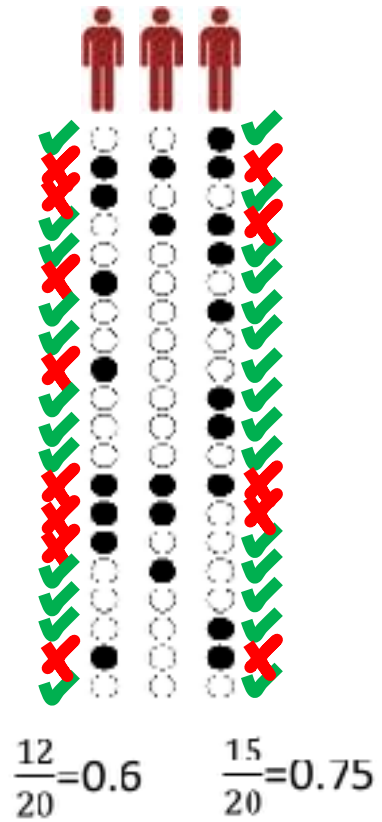
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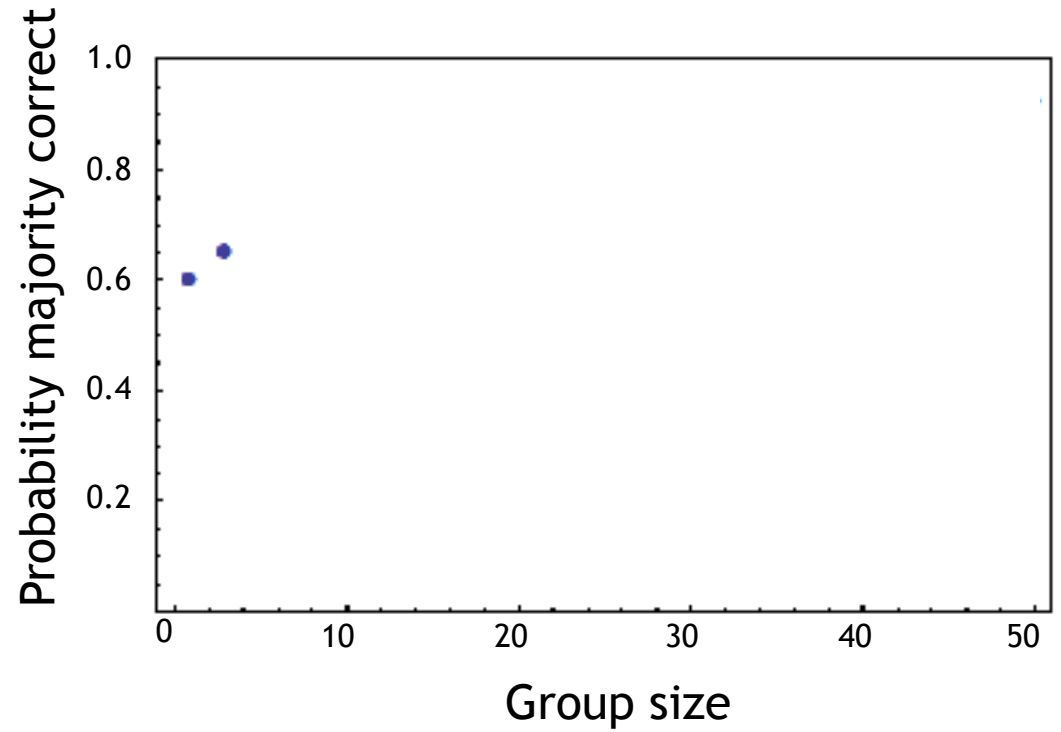
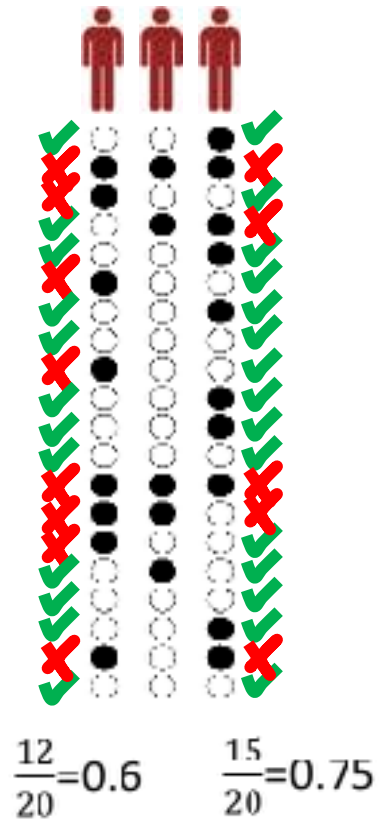
$$\frac{12}{20} = 0.6$$



# Condorcet (1785)

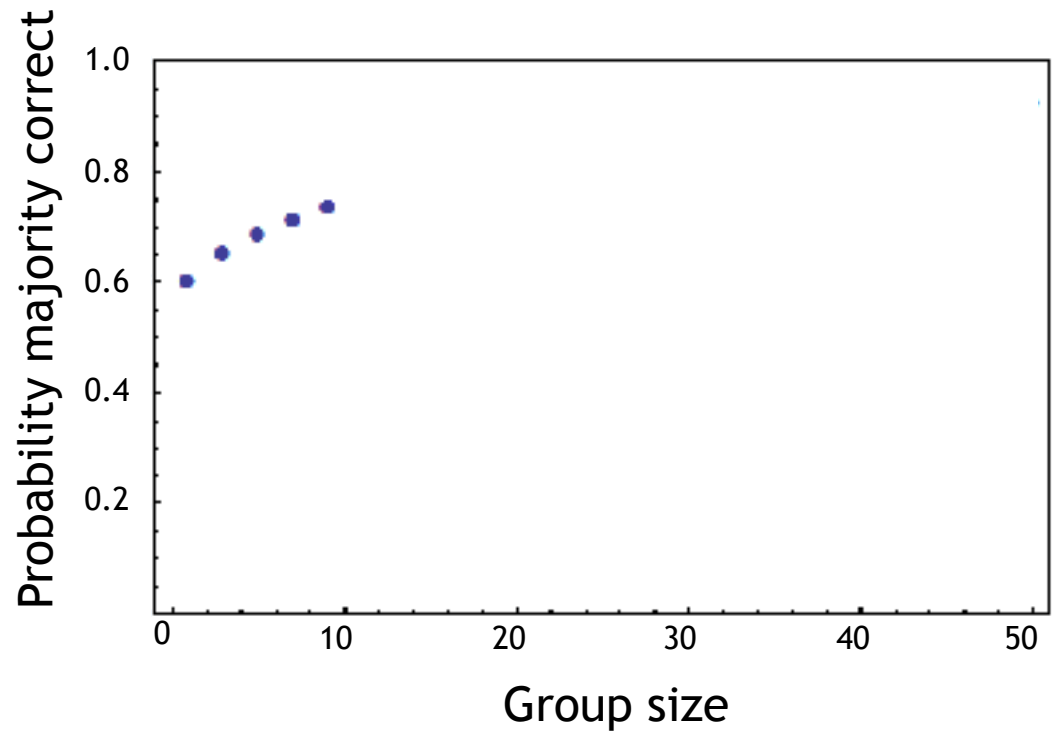
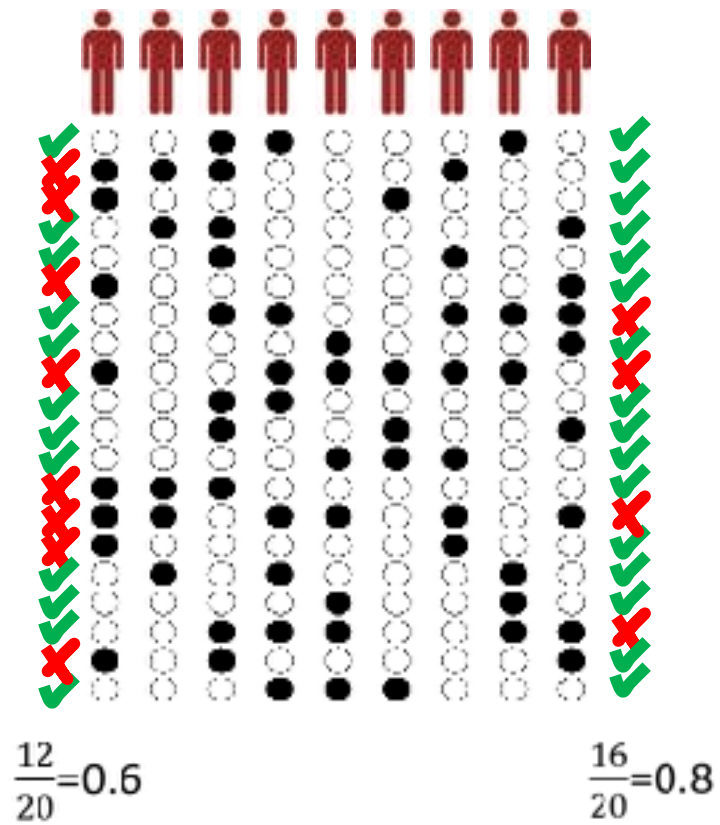


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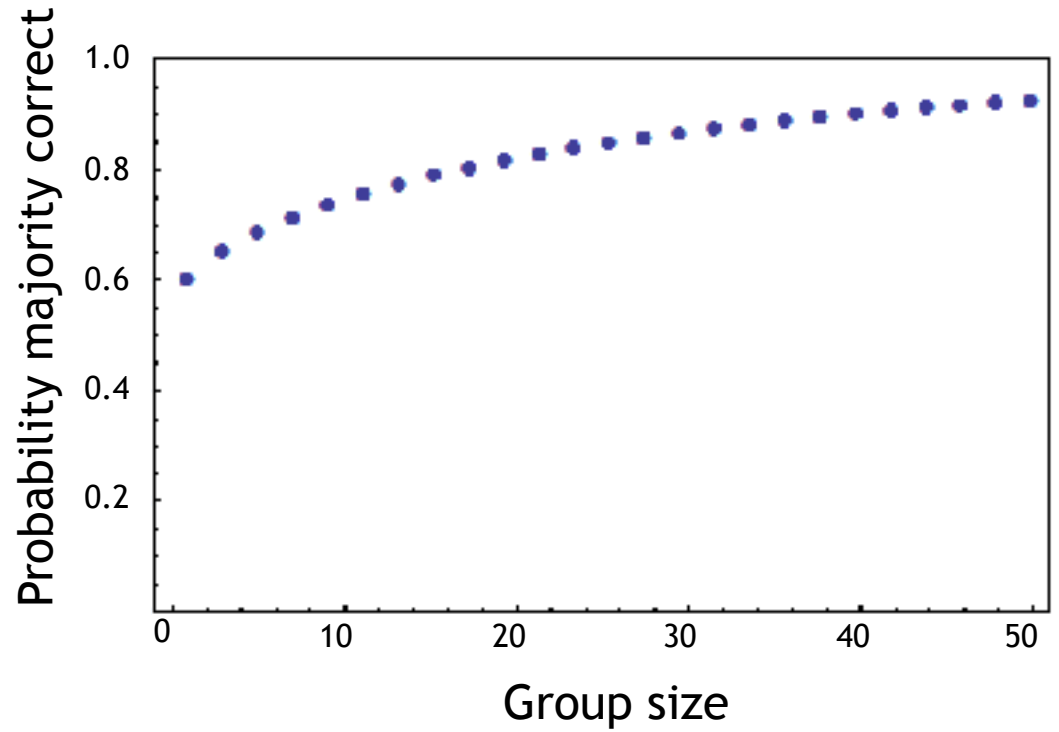
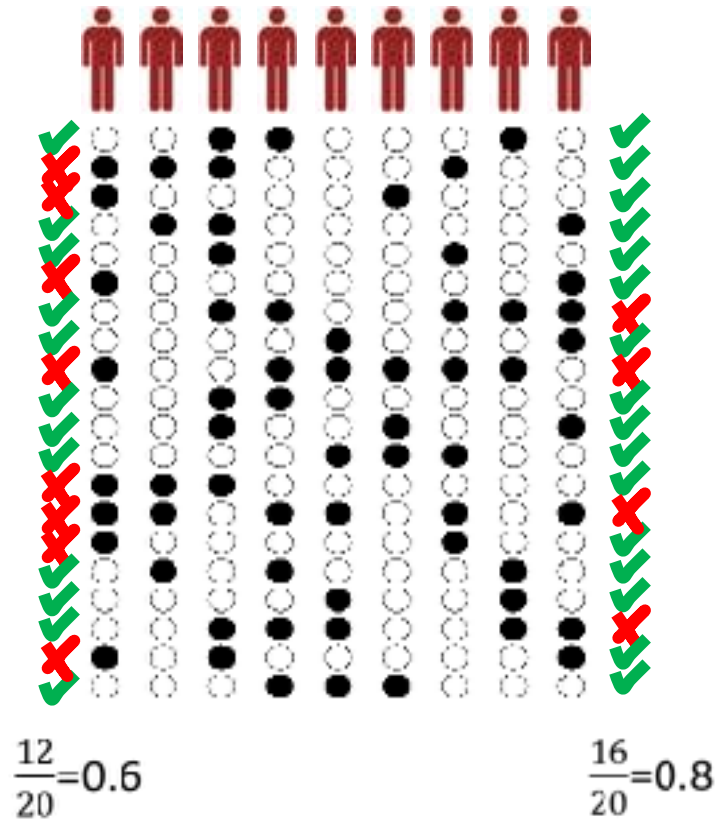




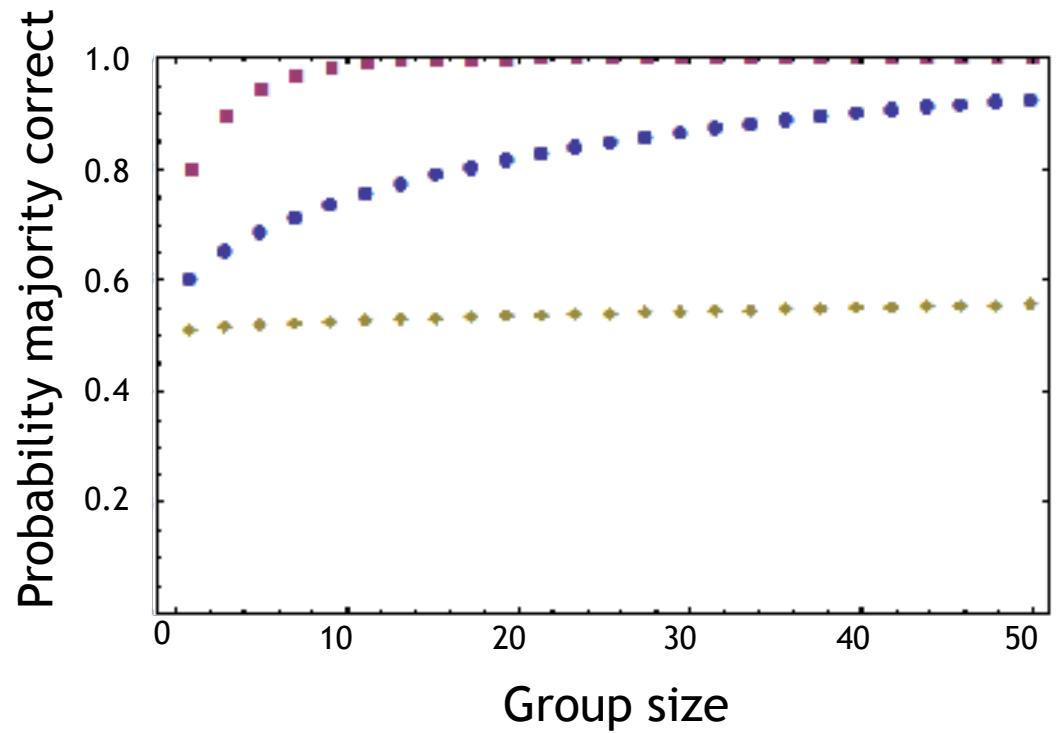
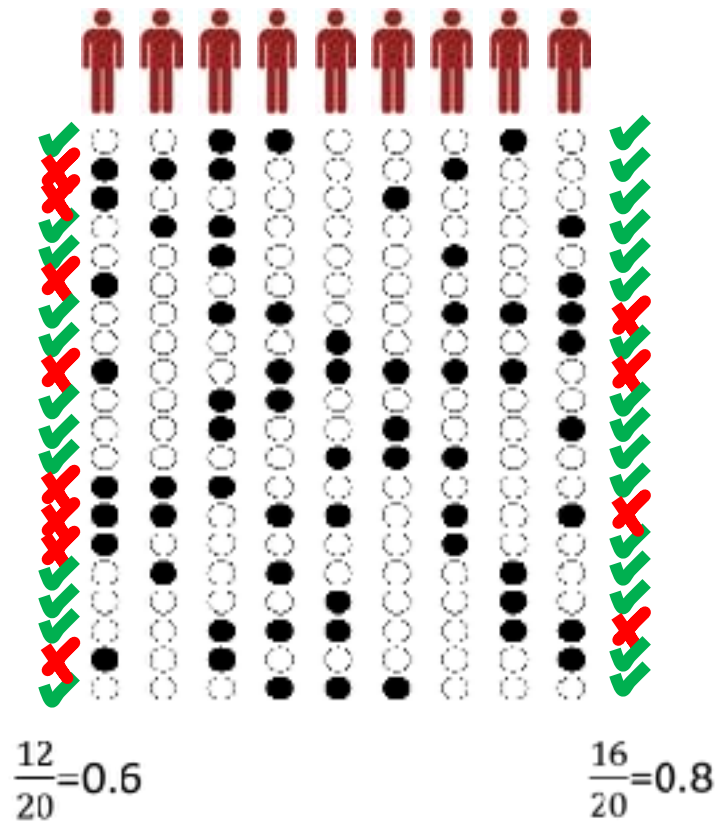
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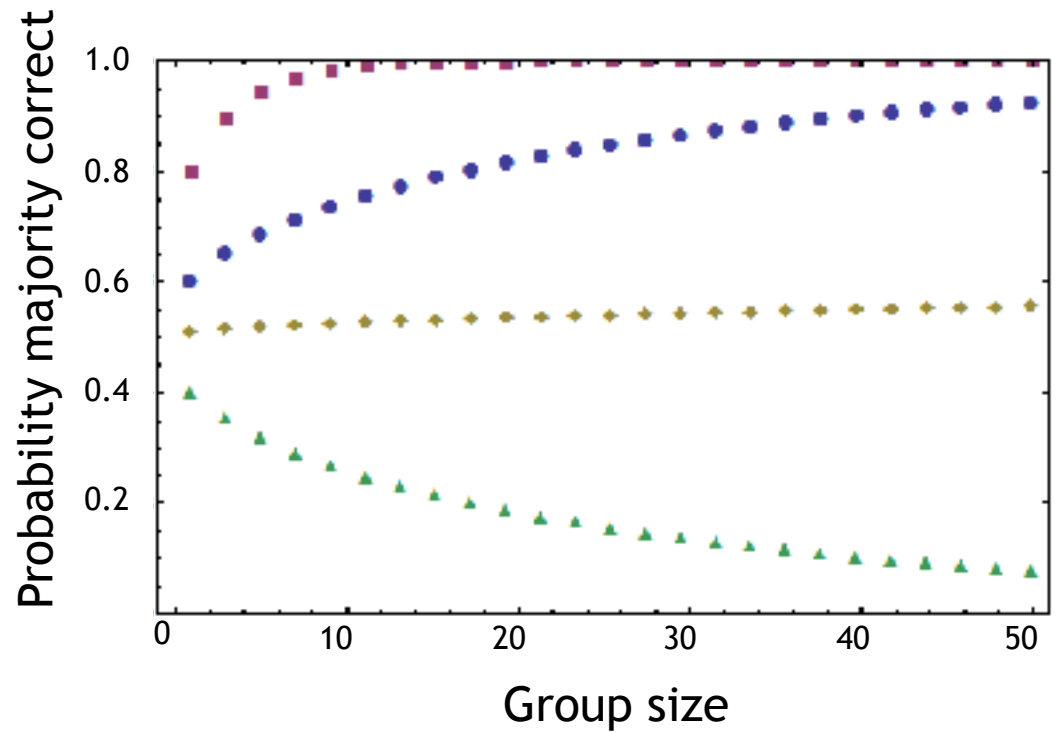
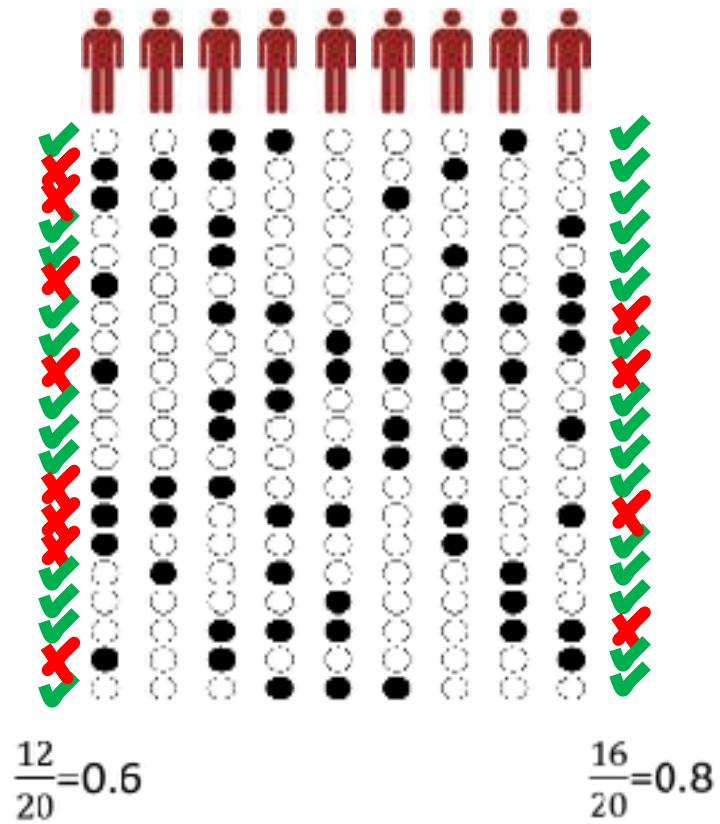
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Is this a proof of Collective Intelligence?

No, it assumes humans to be independent, and we are not because:

1. We receive similar information for many problems
2. We share a common historical/cultural background
3. We share a common cognitive architecture

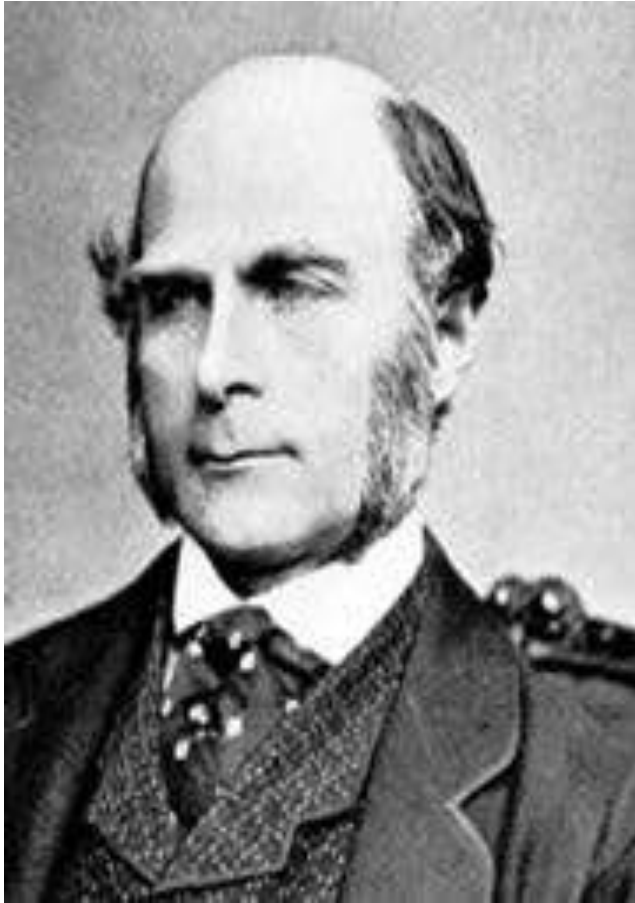
## LESSON 1:

with correlations among individuals, majority voting may NOT be collective intelligence

## LESSON 2:

You also need individuals with  $p > 0.5$

# Galton (1907)



## Francis Galton

### *Vox Populi*

IN these democratic days, any investigation into the trustworthiness and peculiarities of popular judgments is of interest. The material about to be discussed refers to a small matter, but is much to the point.

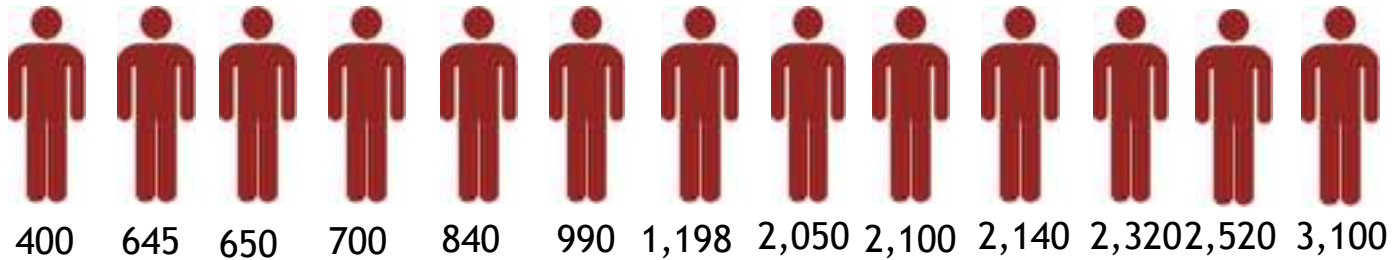


# Galton (1907)

What is the weight of the ox?



Individuals (800) write down their answer independently of each other



Median value (middle observation of ordered list) = 1,198

Real value = 1,207

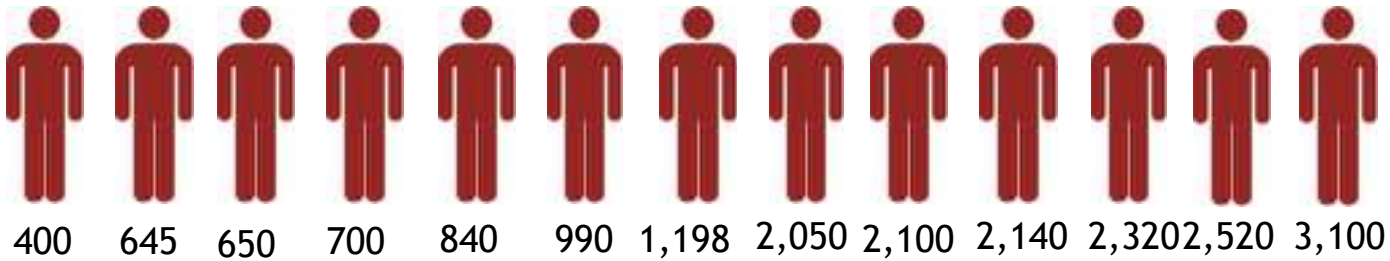
} 1 % error

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Median value (middle observation of ordered list) = 1,198  
Real value = 1,207 } 1 % error

What is the border length between Italy and Switzerland?

Median value = 302  
Real value = 734 } 60 % error



# Galton (1907)

## LESSON 3:

In general, knowledge is not equally distributed in a collective, so doing mean/median is NOT collective intelligence

## **‘Proofs’ that there is collective intelligence in general**

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Galton experiment

Collective error is always smaller

I will show they are NOT proofs, but each will give us a lesson of what is needed for collective intelligence

**How do animals do it? A couple of lessons from animals**

**Can we enhance it with AI?**

## A 'proof' of lower collective error

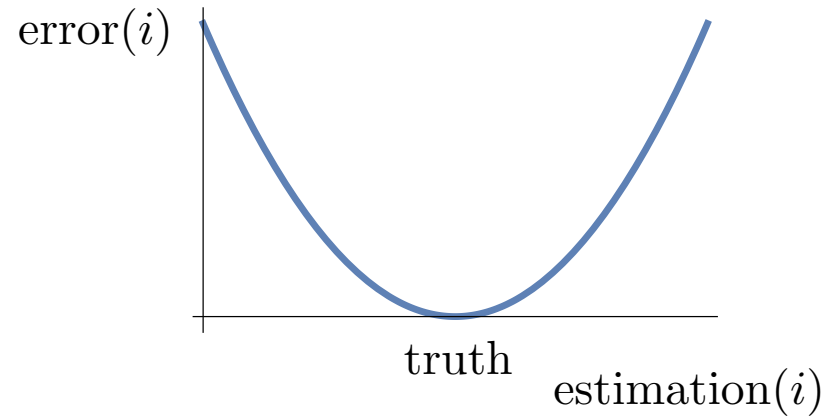
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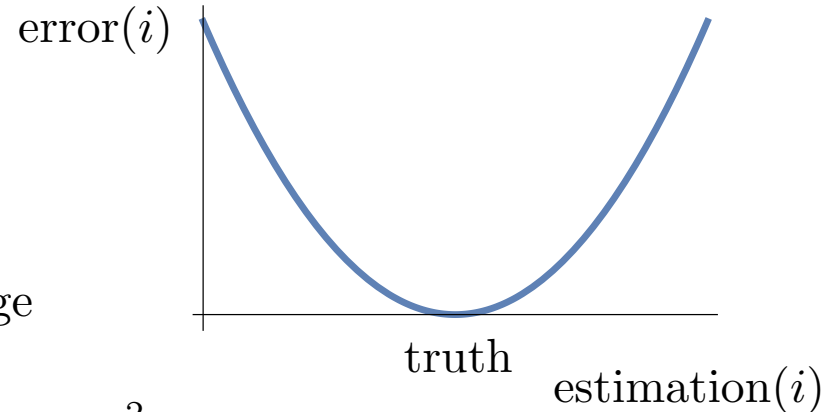
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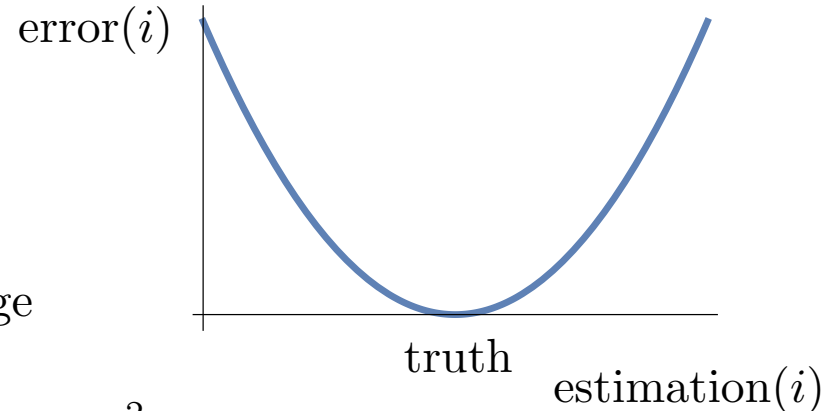
The error each an individual makes is on average

$$\overline{\text{error}(i)} = \frac{1}{N} \sum_{i=1}^N (\text{estimation}(i) - \text{truth})^2$$

The collective estimation is simply the average estimation

$$\text{collective estimation} = \frac{1}{N} \sum_{i=1}^N \text{estimation}(i)$$

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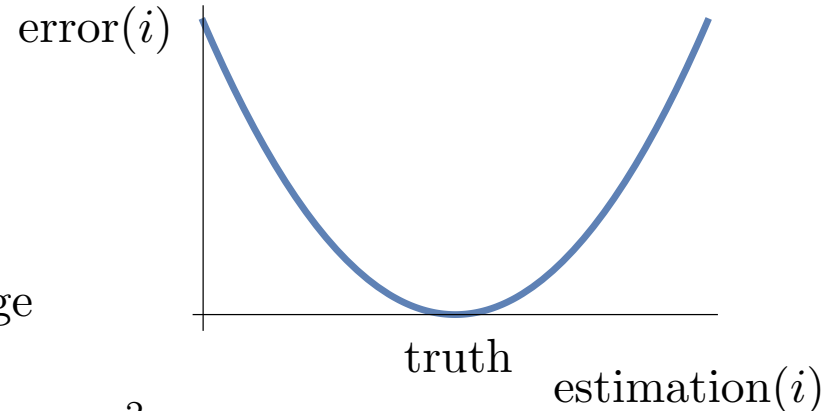
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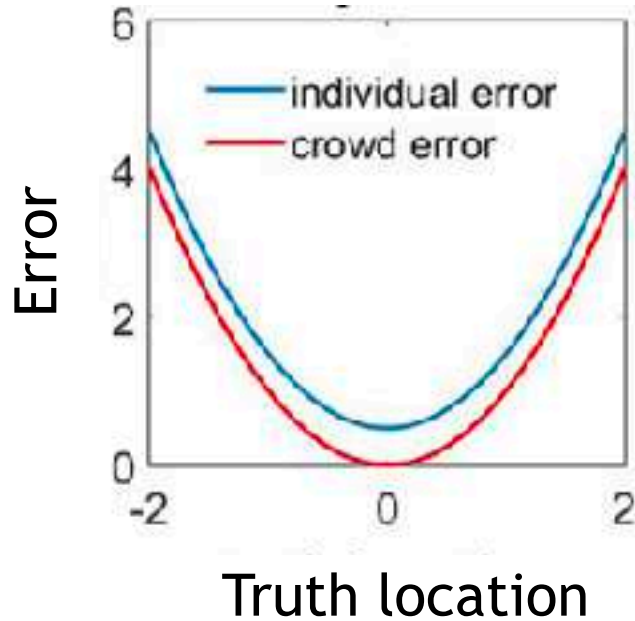
$$\text{collective error} = (\text{collective estimation} - \text{truth})^2$$

**Theorem:** The collective estimation has on average less or the same error than a randomly picked individual

$$\text{collective error} \leq \overline{\text{error}(i)}$$



Example: 5 people estimate: -1,-0.5,0,0.5,1

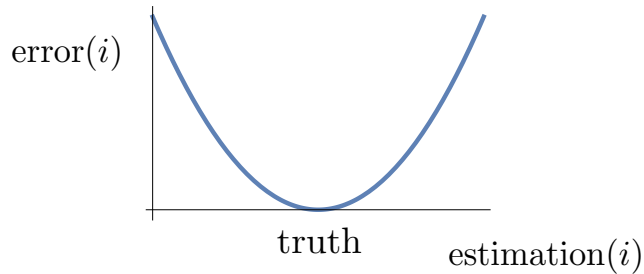


Crowd better on average than an individual independently of truth

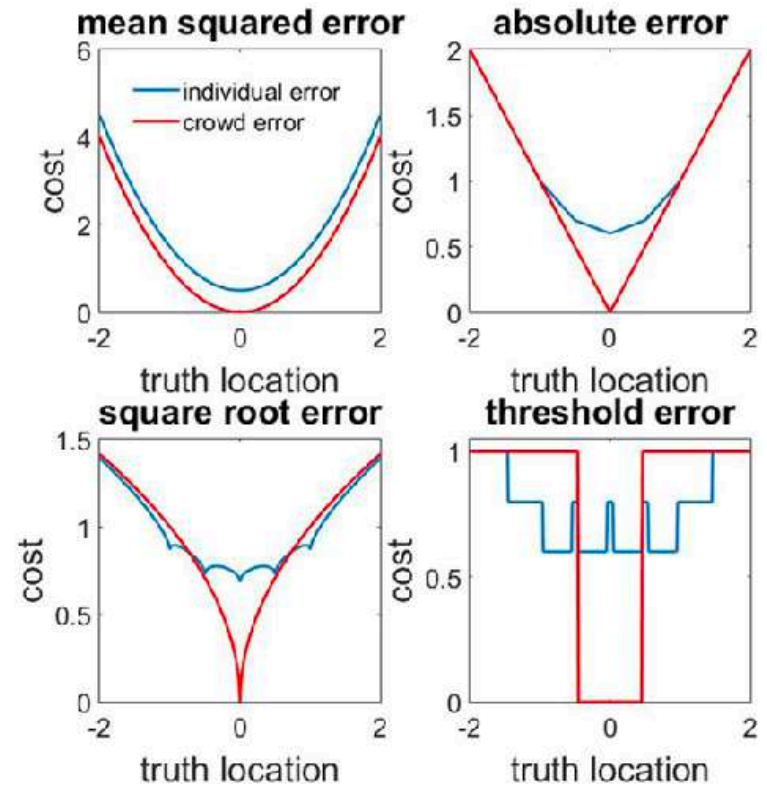
Example 2: Use the length of our noses to measure the height of Eiffel Tower. Again, the crowd is better!



What happens is that the error is assumed to be quadratic (because we are lazy intellectuals) but *real* errors need not be



For other errors the result is different



#### LESSON 4:

Errors in a collective can be smaller or higher than individual error depending on the error function

## BIG LESSON:

There is no magic bullet for Collective Intelligence

Instead, it needs to be incentivised by methods that

Allow those with knowledge to have more weight ([anti-Galton](#))

Correlations among individuals should be such the collective listens to those that at each point in time have the required knowledge to solve the task ([anti-Condorcet](#))

Understand the cost incurred by mistakes ([anti-Theorem](#))

## **‘Proofs’ that there is collective intelligence**

Condorcet jury model

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Collective error is always smaller

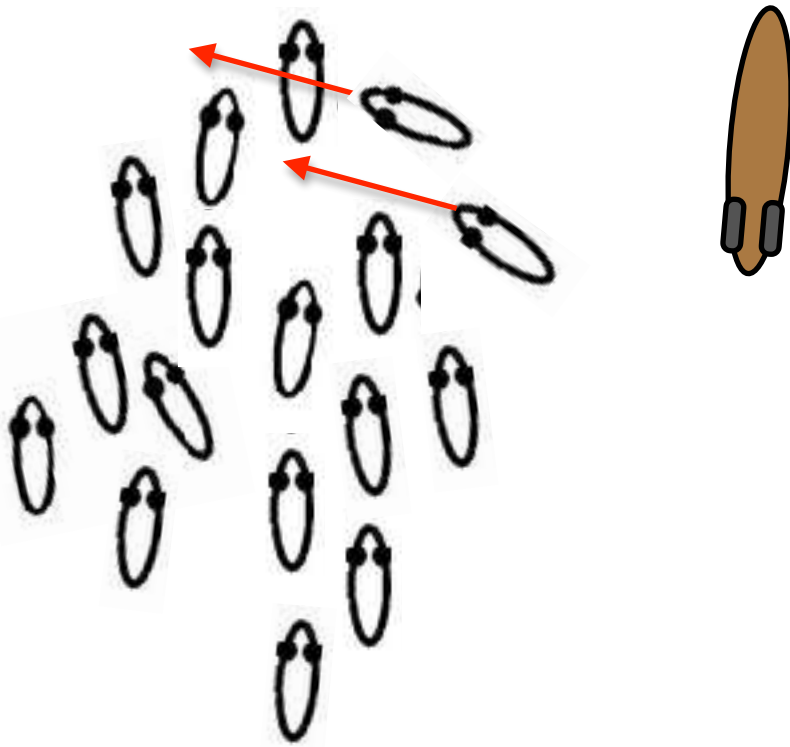
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**How do animals do it? A couple of lessons from animals**

**Can we enhance it with AI?**



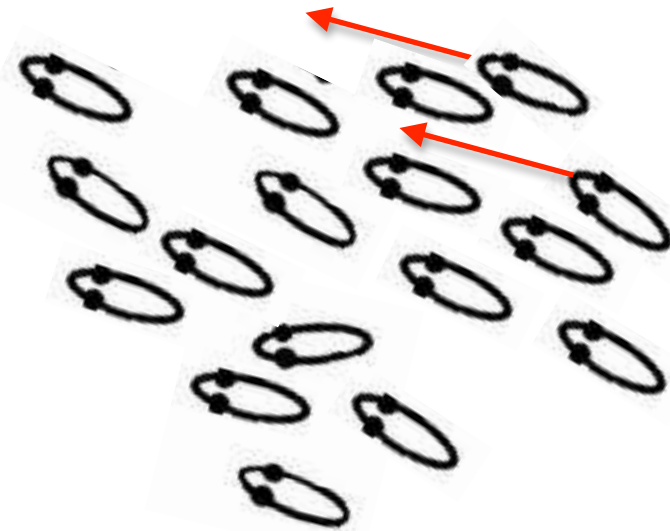
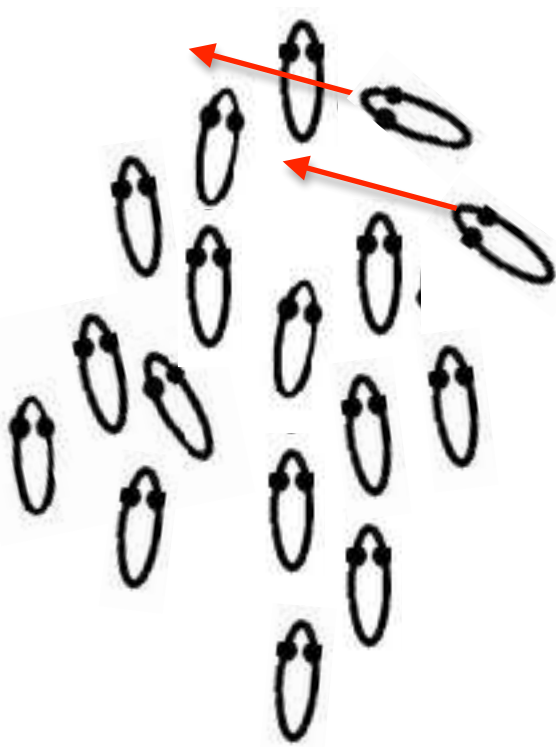
Arganda, Perez-Escudero & de Polavieja, *PNAS* (2012)  
Perez-Escudero et al, *Nat. Methods* (2014)  
Hinz & de Polavieja, *PNAS* (2017)  
Perez-Escudero & de Polavieja, *Interface* (2017)  
Laan, Gil de Sagredo & de Polavieja, *Proc. Roy. Soc.* (2017)  
Vicente-Page, Perez-Escudero & de Polavieja, *J. Theor. Ecology* (2018)



Those with knowledge are copied by the rest (instead of using a majority vote)

Copy takes place more likely when those with knowledge **confidently** show it and those that do not have do not look as if they had it

- (a) high velocity/acceleration changes & straight paths
- (b) More so if  $>1$  individual with properties in (a)



Those with knowledge are copied by the rest (instead of using a majority vote)

Copy takes place more likely when those with knowledge **confidently** show it and those that do not have do not act as if they had it

- (a) high velocity/acceleration changes & straight paths
- (b) More relevant if  $>1$  individual with properties in (a)

## Animal collectives are more intelligent when:

Individuals with knowledge express it to the collective

The others can recognise who has the knowledge and use it

The diversity of knowledge to solve a task is present in the collective

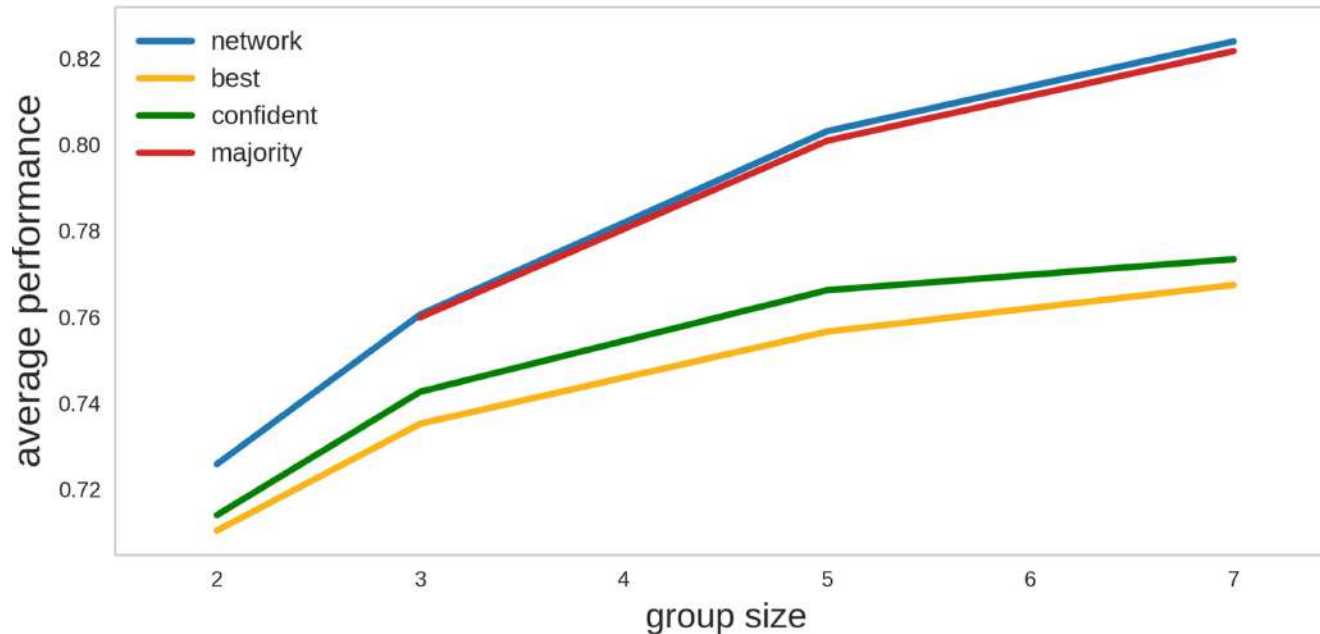
## Does AI help?

$N$  doctors ( $N=2,3, 5$  or  $7$ ) diagnose cancer/no cancer from image and also give confidence level and the % of successes until then

The strategy of choosing the best (higher % correct) or more confident is suboptimal. The majority opinion is quite good (because most doctors are reasonably good).

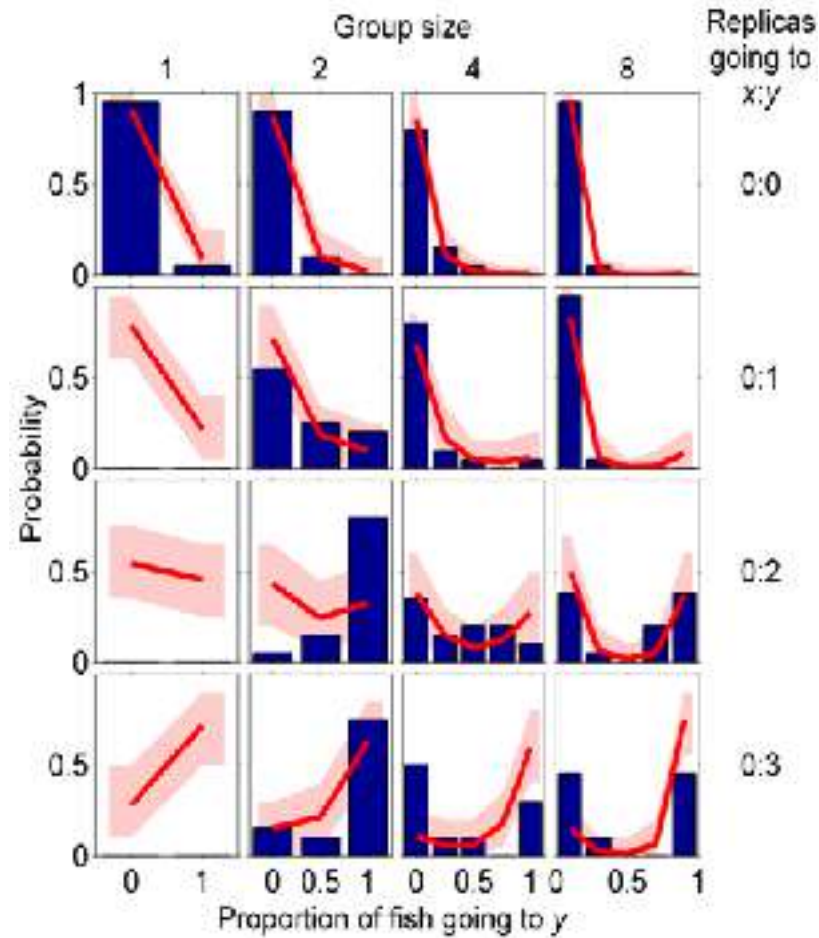
The network trained to combine doctors opinions, confidence levels and % correct is slightly better (significant for  $N=5$  and  $N=7$ )

**Collectives + AI** might be a good method to extract collective intelligence, specially when knowledge is only in few individuals

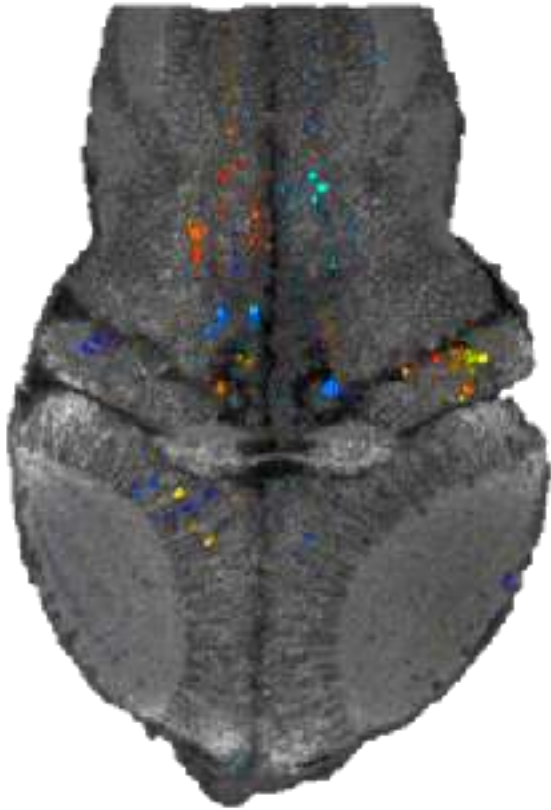








# What are our theories based upon?



Individuals use **inference**

Individuals learn by **rewards**

Individuals **control** their behavior according to some policies

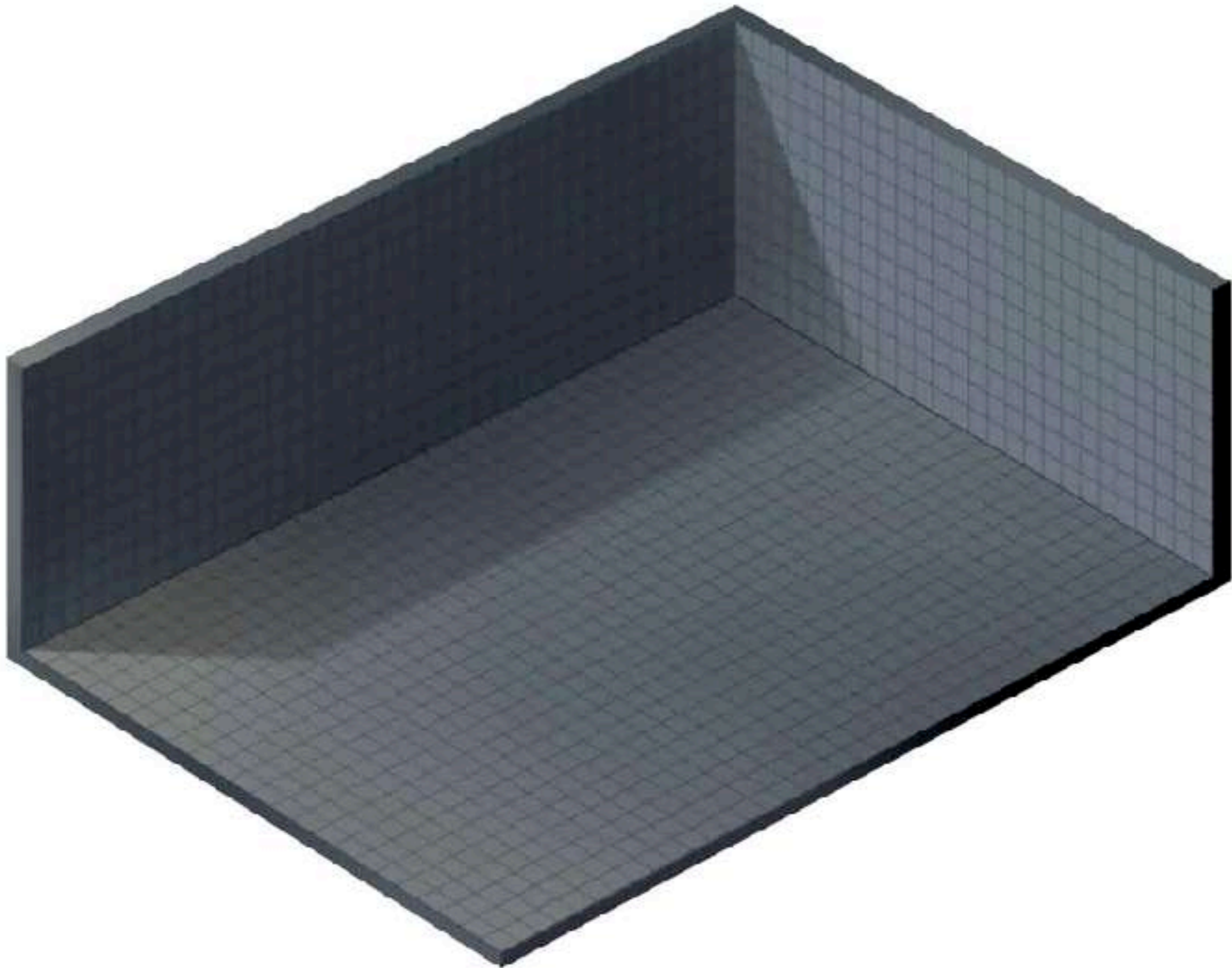
Individuals use **heuristic** rules

Individuals are a **neuronal network** coupled to a skeletal system

Brains generate decisions from ambiguous information



$$P(\text{fish} \mid \text{plants})$$



# Brains generate decisions from ambiguous information



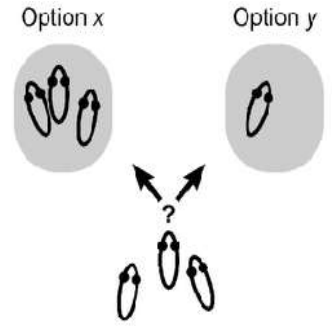
$$P(\text{fish} \mid \text{plants})$$



$$P(\text{fish} \mid \text{plants}, \text{school})$$

$P(Y|C, B)$

$Y$  'y is the best option'  
 $C$  'private information'  
 $B$  'behaviors of others'



$$P(X|C, B) = 1 - P(Y|C, B)$$

$$P(Y|C, B) = \frac{P(B|Y, C)P(Y|C)}{P(B|X, C)P(X|C) + P(B|Y, C)P(Y|C)}$$

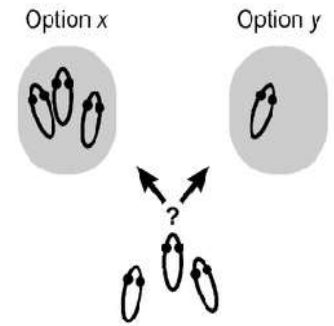
$$P(Y|C, B) = \frac{1}{1 + aS}$$

$$a = \frac{P(X|C)}{P(Y|C)}$$

$$S = \frac{P(B|X, C)}{P(B|Y, C)}$$

$$S = \frac{P(B|X, C)}{P(B|Y, C)}$$

$$B = \{b_i\}$$



Assuming focal agent does not use correlations among others  
(see our PCB 2011 without this assumption)

$$P(B|Y, C) = Z \prod_{i=1}^N P(b_i|Y, C) \quad S = \prod_{i=1}^N \frac{P(b_i|X, C)}{P(b_i|Y, C)}$$

Instead of behaviours  $b_i$  consider  $\beta_x$  as ‘going to  $x$ ’ and  $\beta_y$  ‘going to  $y$ ’  
with  $n_x$  animals going to  $x$  and  $n_y$  animals going to  $y$

$$S = s_x^{n_x} s_y^{n_y} \quad s_x = \frac{P(\beta_x|X, C)}{P(\beta_x|Y, C)} \quad s_y = \frac{P(\beta_y|X, C)}{P(\beta_y|Y, C)}$$



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In the case of a symmetric set-up (both options identical)

$$s_x = \frac{1}{s_y} \equiv s$$

$$S = s_x^{n_x} s_y^{n_y} = s^{n_x} s^{-n_y} = s^{-(n_y - n_x)}$$

$$P(Y|C, B) = \frac{1}{1 + aS}$$

$$P(Y|C, B) = \frac{1}{1 + aS^{-(n_y - n_x)}}$$

That is the estimation part of the derivation.

Now we need a decision-rule using this estimation.

The pure one would be:

Choose  $y$  when  $P(Y|C, B) > P(X|C, B)$ . Otherwise, choose  $x$

This pure rule does not correspond to data.

Instead we can add noise or use the softer parameter-free version  
(probability matching rule)

$$P_y = P(Y|C, B) = \frac{1}{1 + aS^{-(n_y - n_x)}}$$

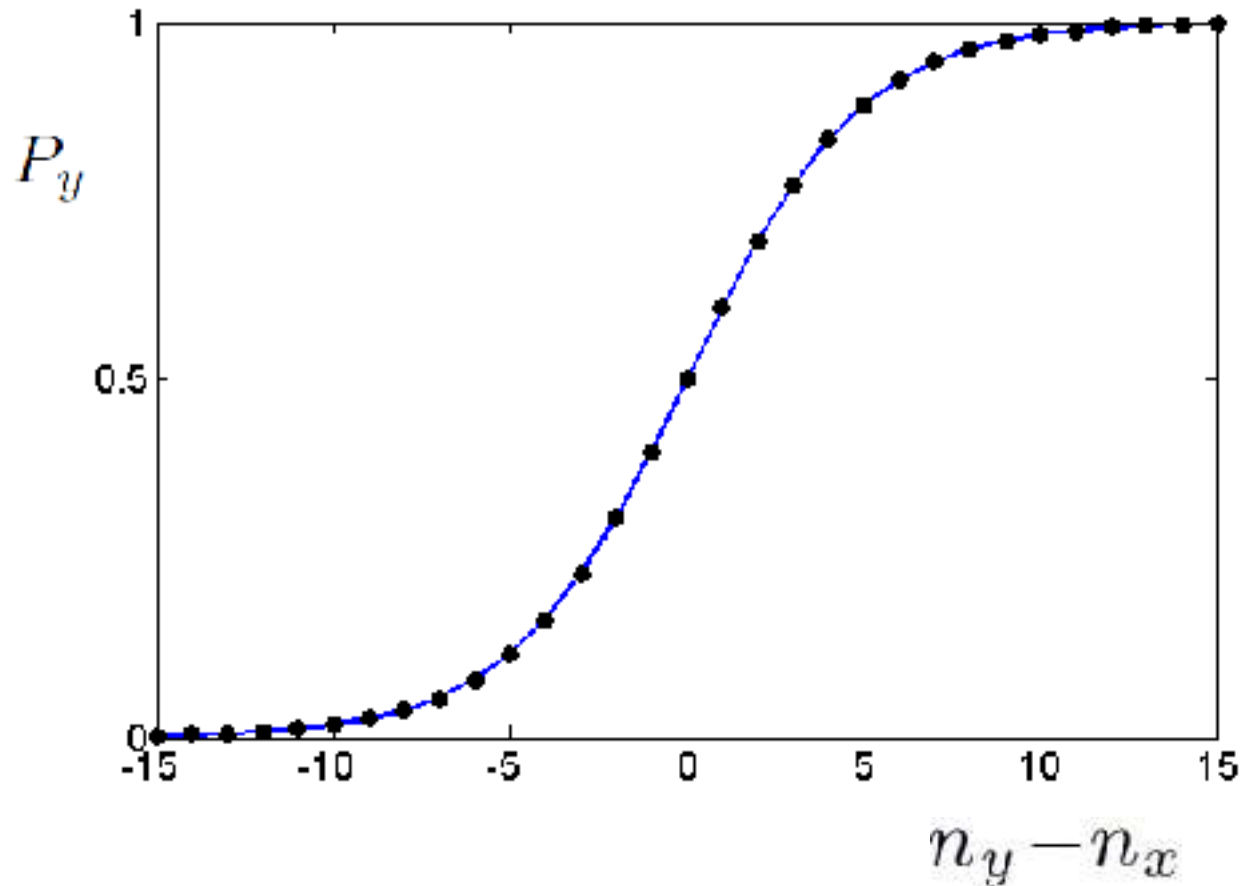
Option x

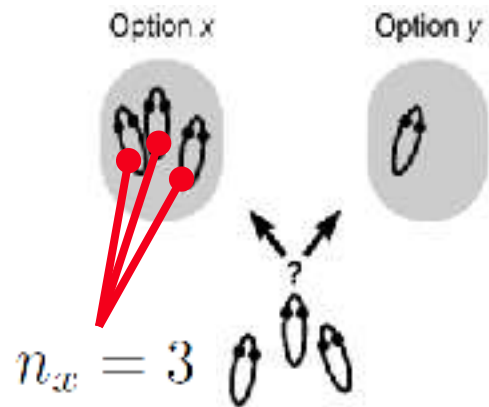


Option y

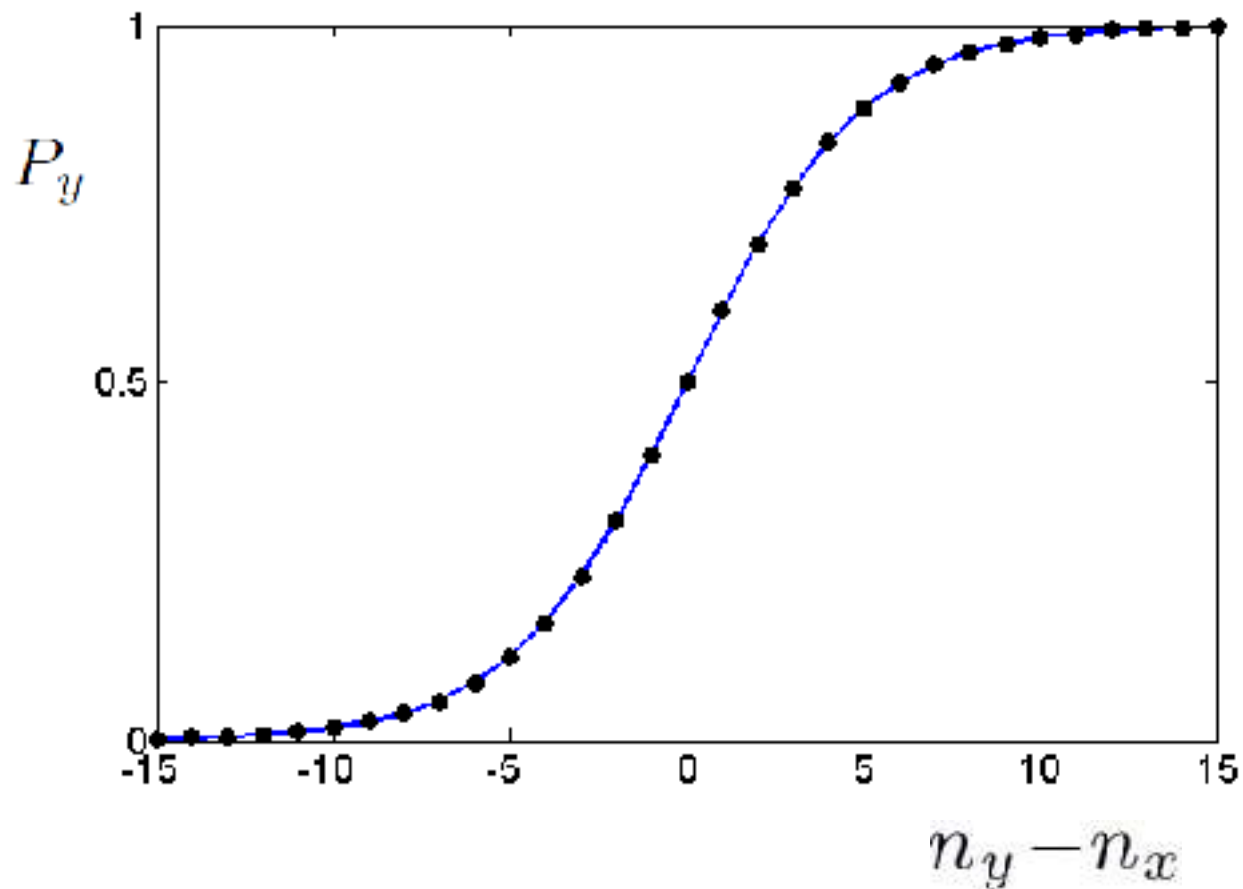


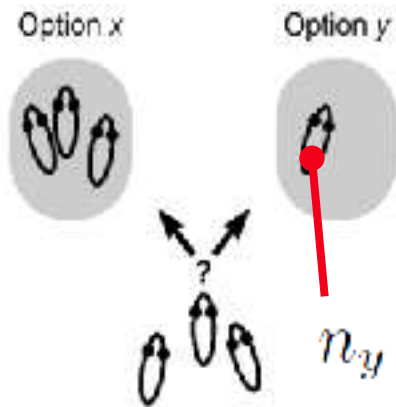
$$P_y = \frac{1}{1 + aS^{-(n_y - n_x)}}$$



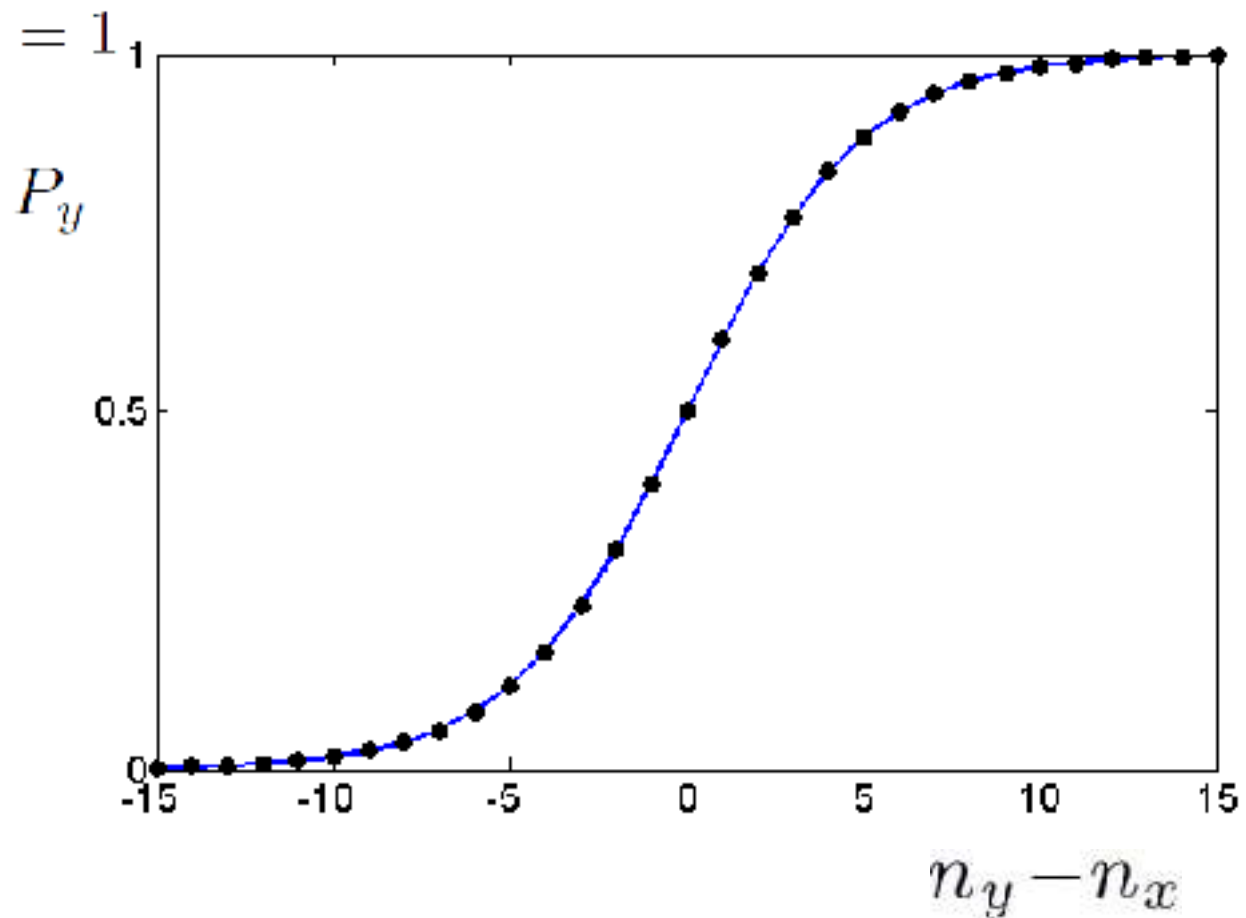


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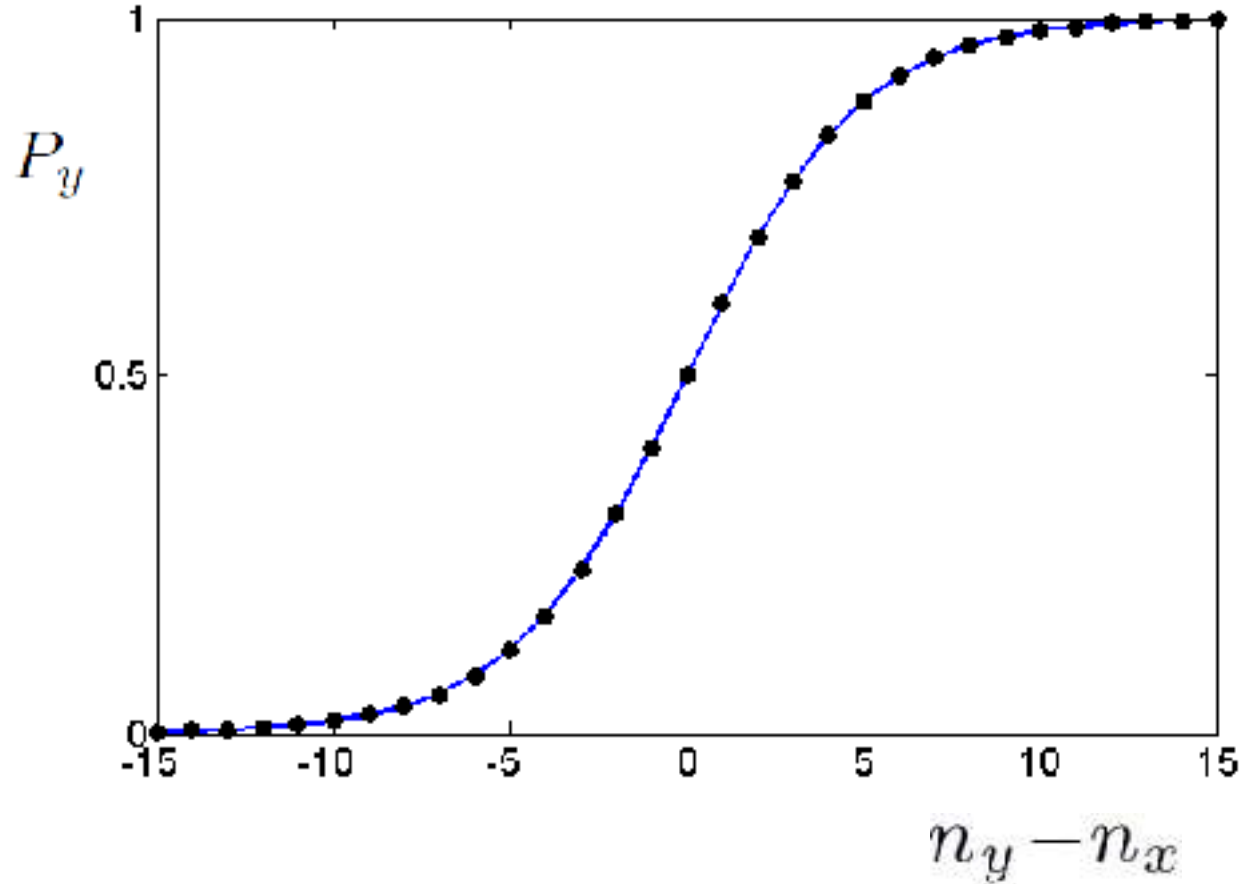
Option y



$$P_y = \frac{1}{1 + a s^{-(n_y - n_x)}}$$

$$a = \frac{P(X|C)}{P(Y|C)}$$

How private info alone tells George which option is best



Option x

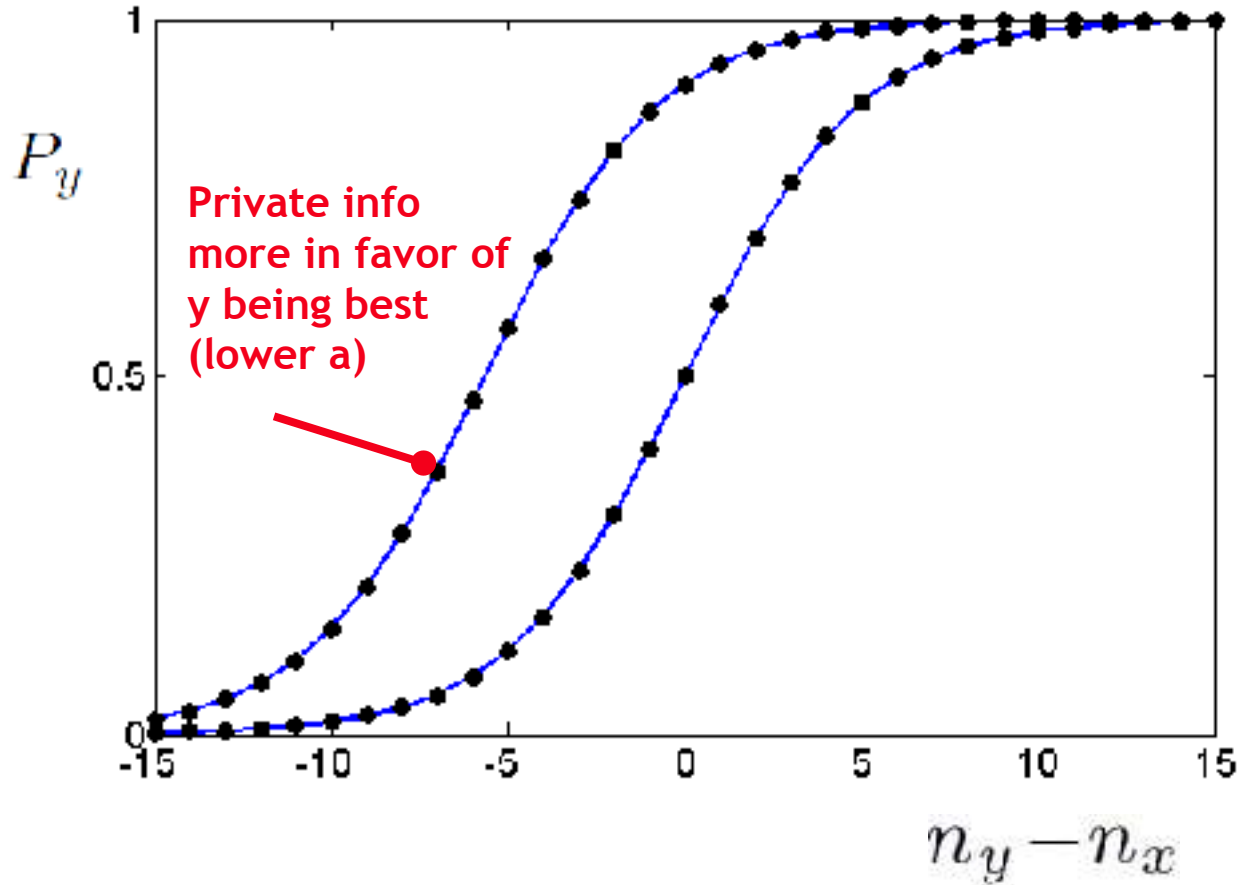
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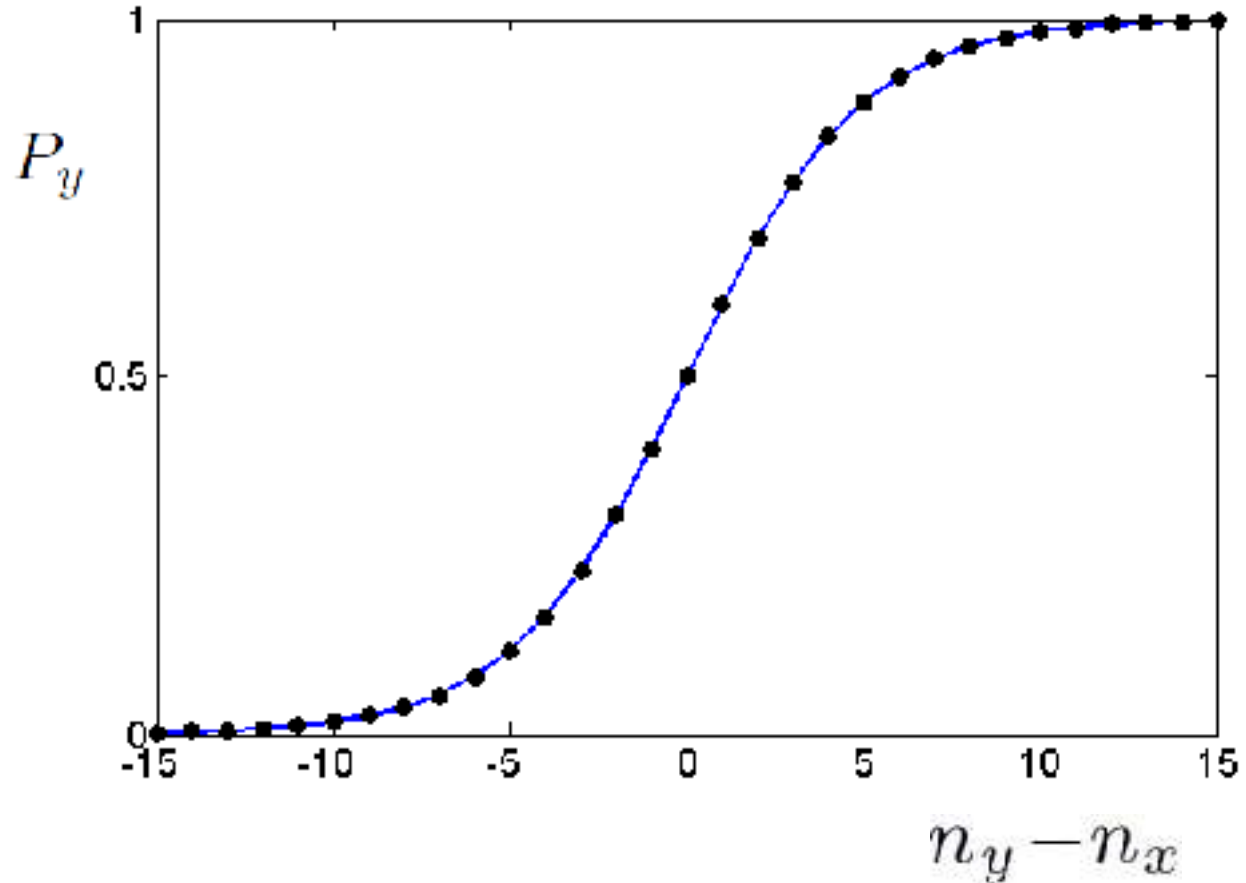
Option y



$$P_y = \frac{1}{1 + a s^{-(n_y - n_x)}}$$

$$s = \frac{P(\beta_y | Y, C)}{P(\beta_y | X, C)}$$

How reliably  
one of George's friends  
chooses one option  
when it is the best





Option x

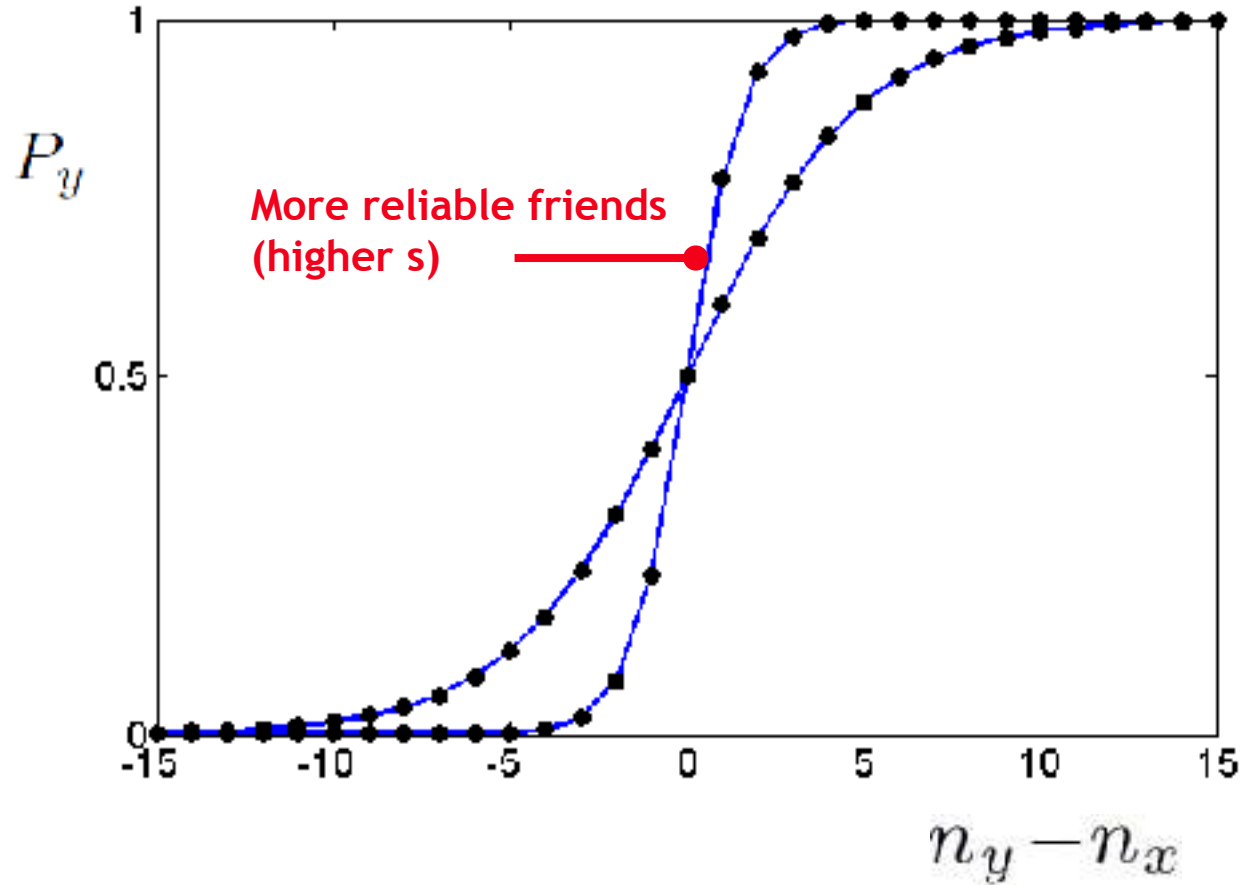
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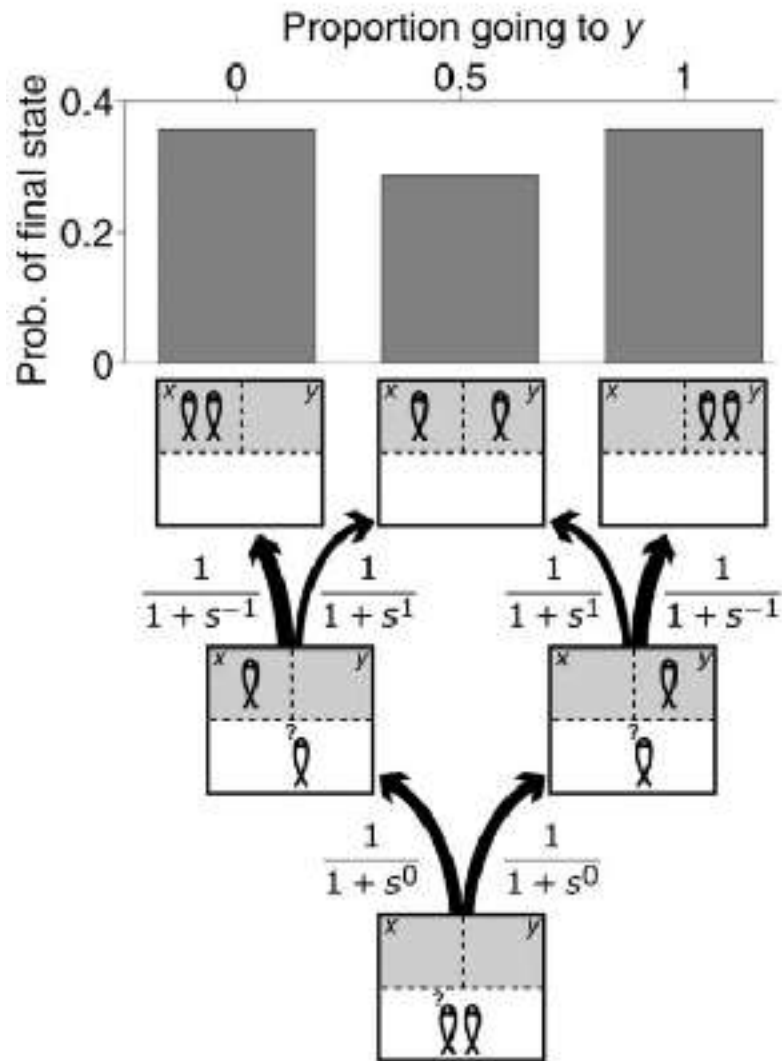


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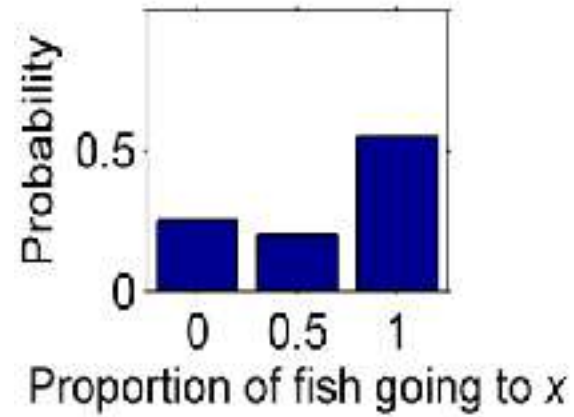
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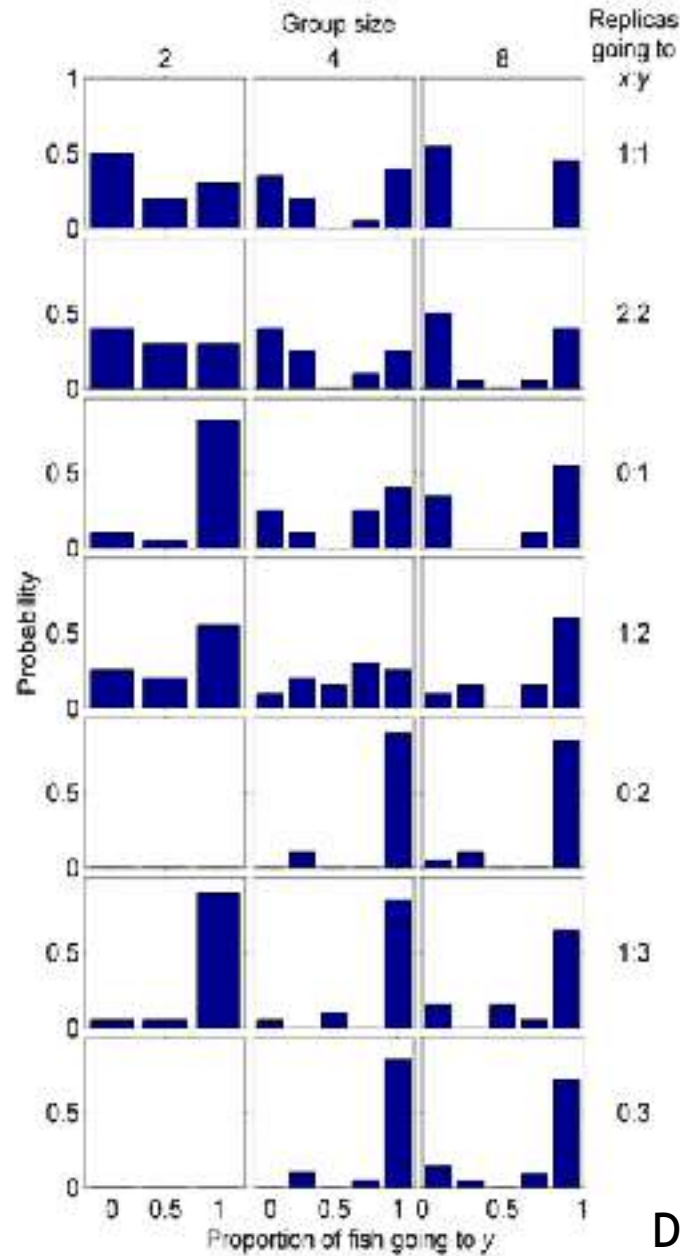




# Test in sticklebacks

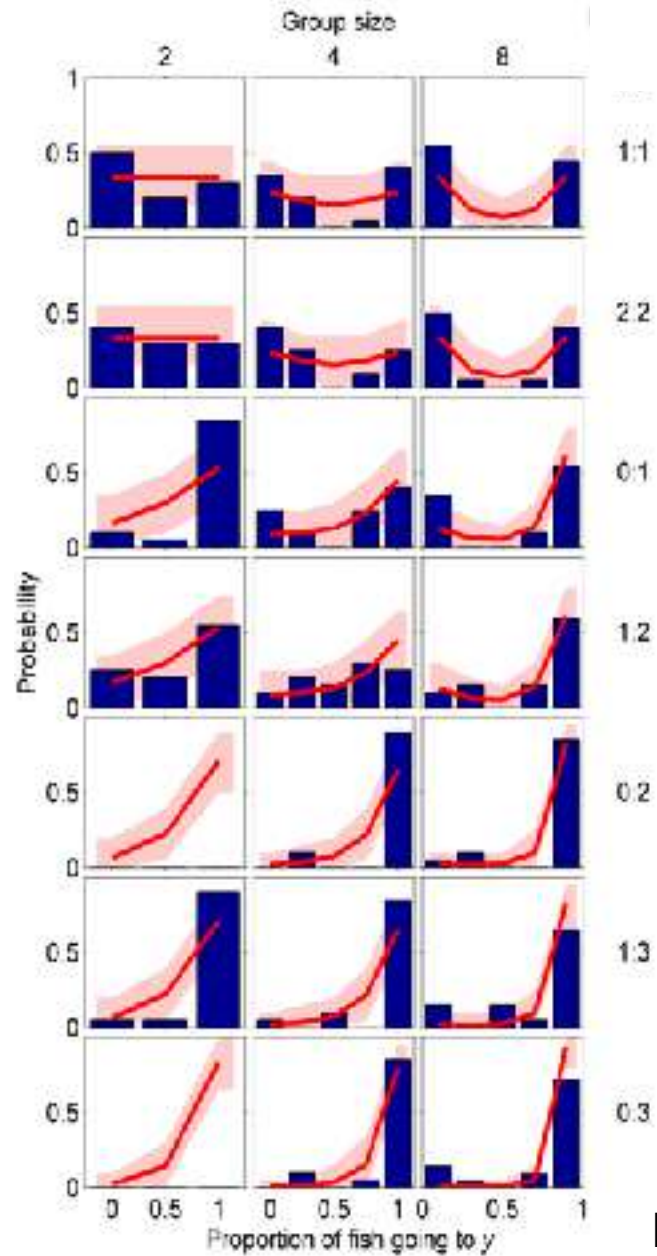


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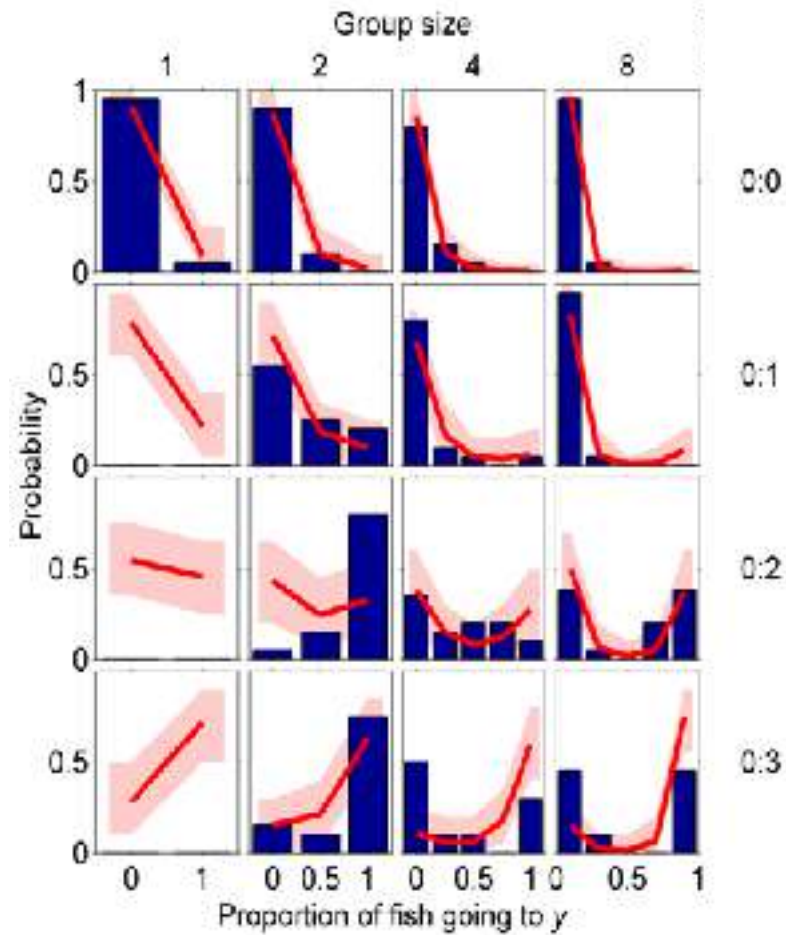
Data from Ward *et al.* (2005)

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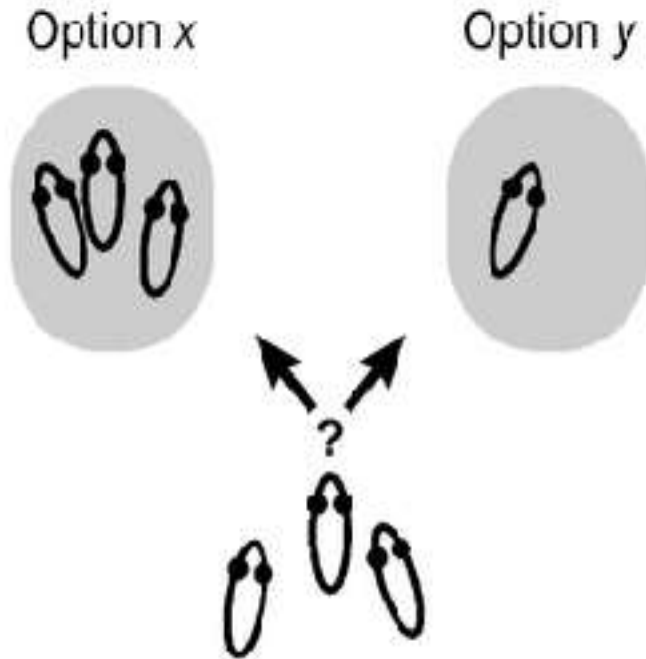
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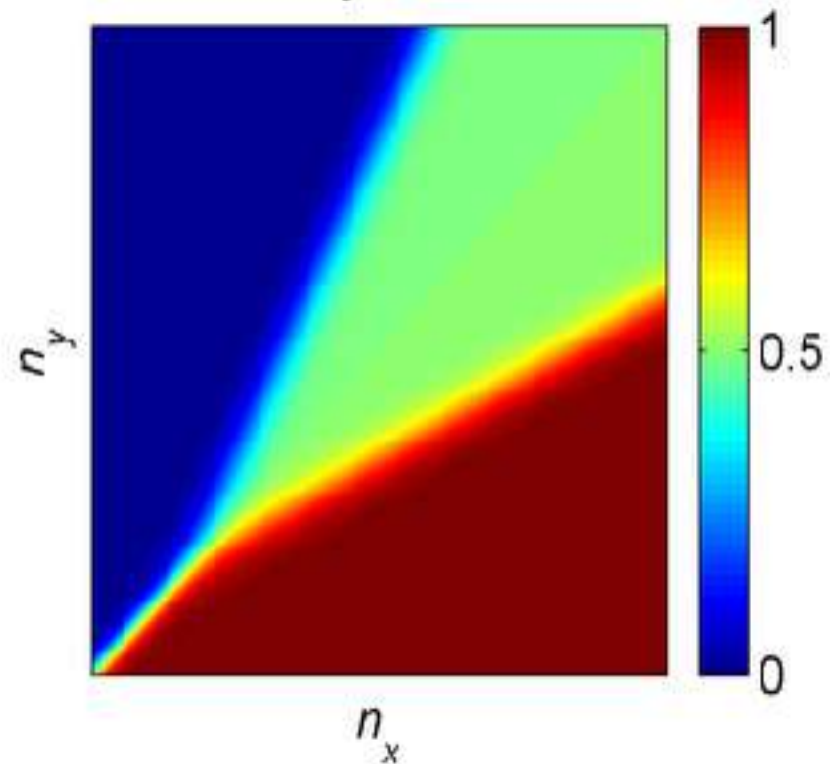
Data from Ward *et al.* (2005)

Before when one option is the best then other is the worst

Now the two options can in principle be good or bad

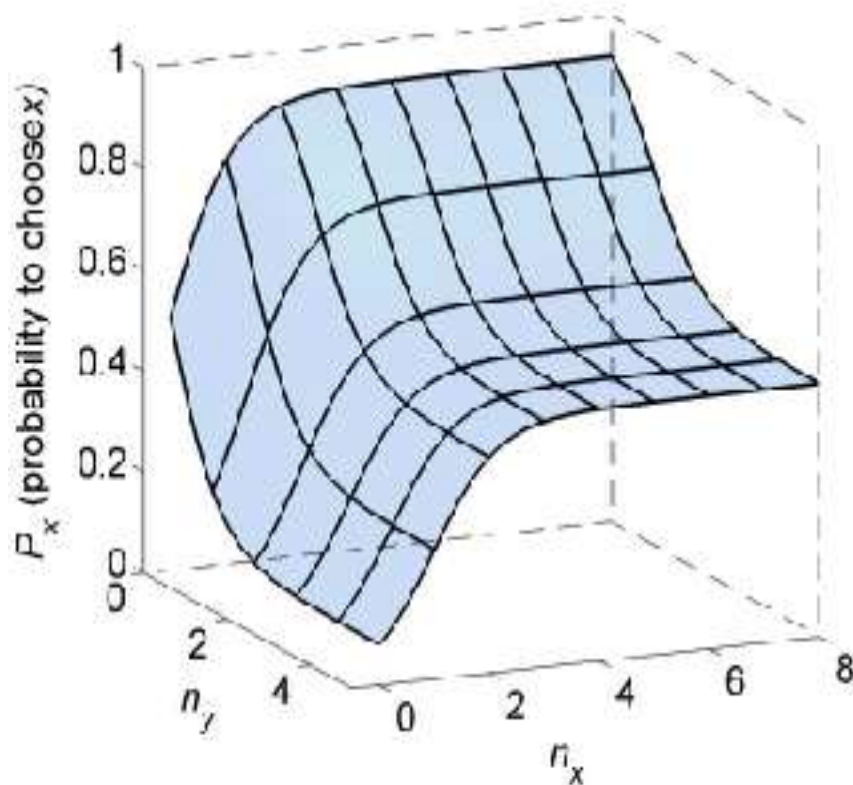


$$P_x = \left( 1 + \frac{1 + a.s^{-(n_x - kn_y)}}{1 + a.s^{-(n_y - kn_x)}} \right)^{-1}$$



# Decisions in zebrafish match the theory

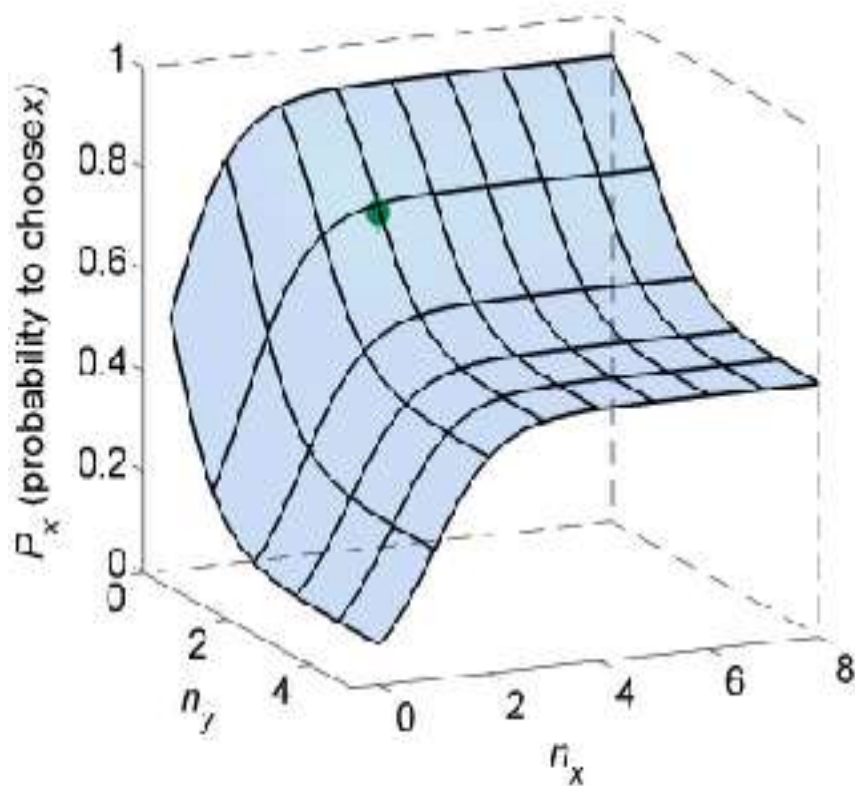
$$P_x = \left( 1 + \frac{1 + aS^{-(n_x - kn_y)}}{1 + aS^{-(n_y - kn_x)}} \right)^{-1}$$





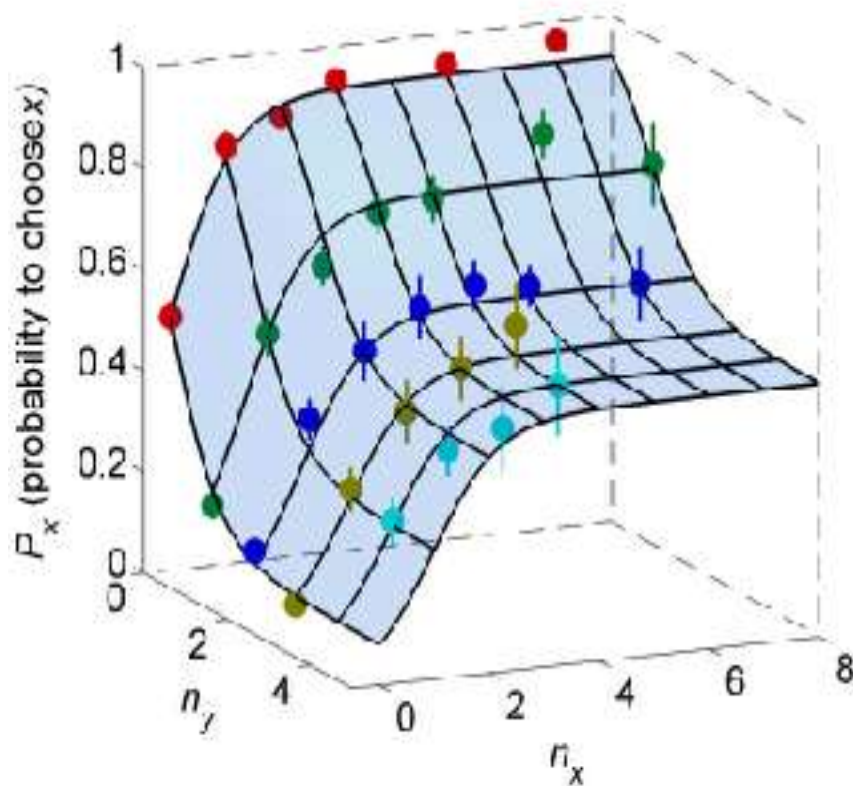
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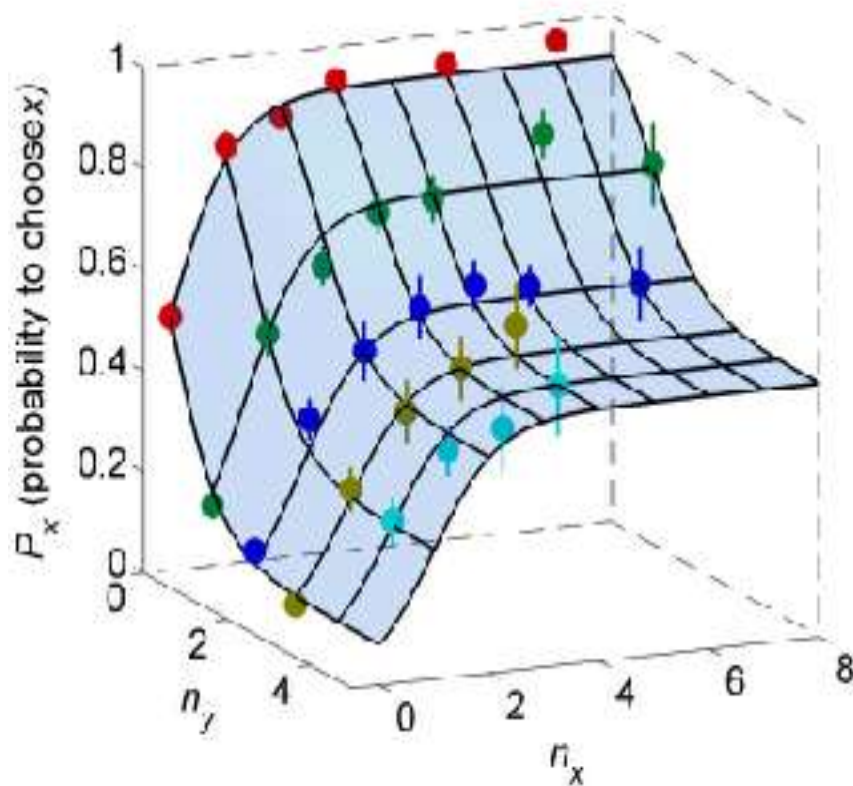
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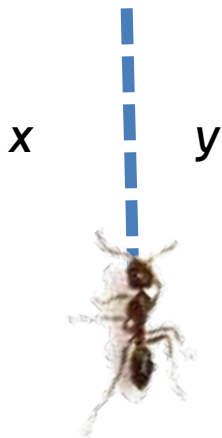
# Decisions in zebrafish match the theory

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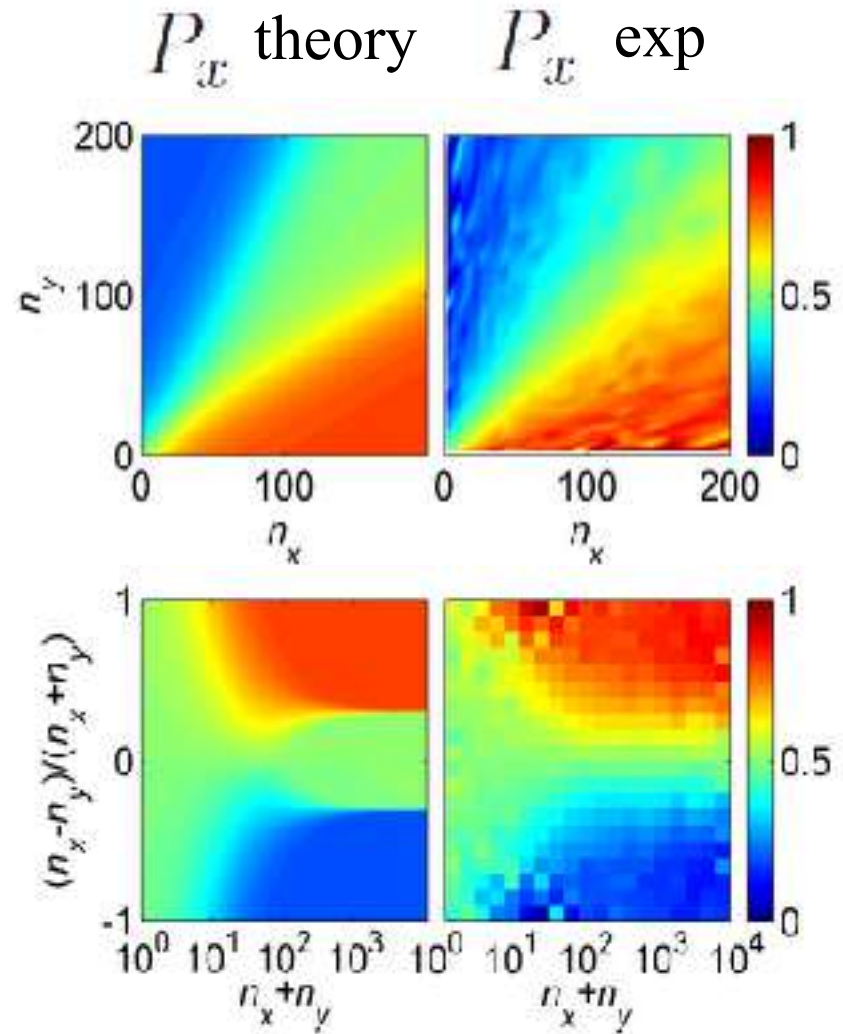


# Also good match with ants

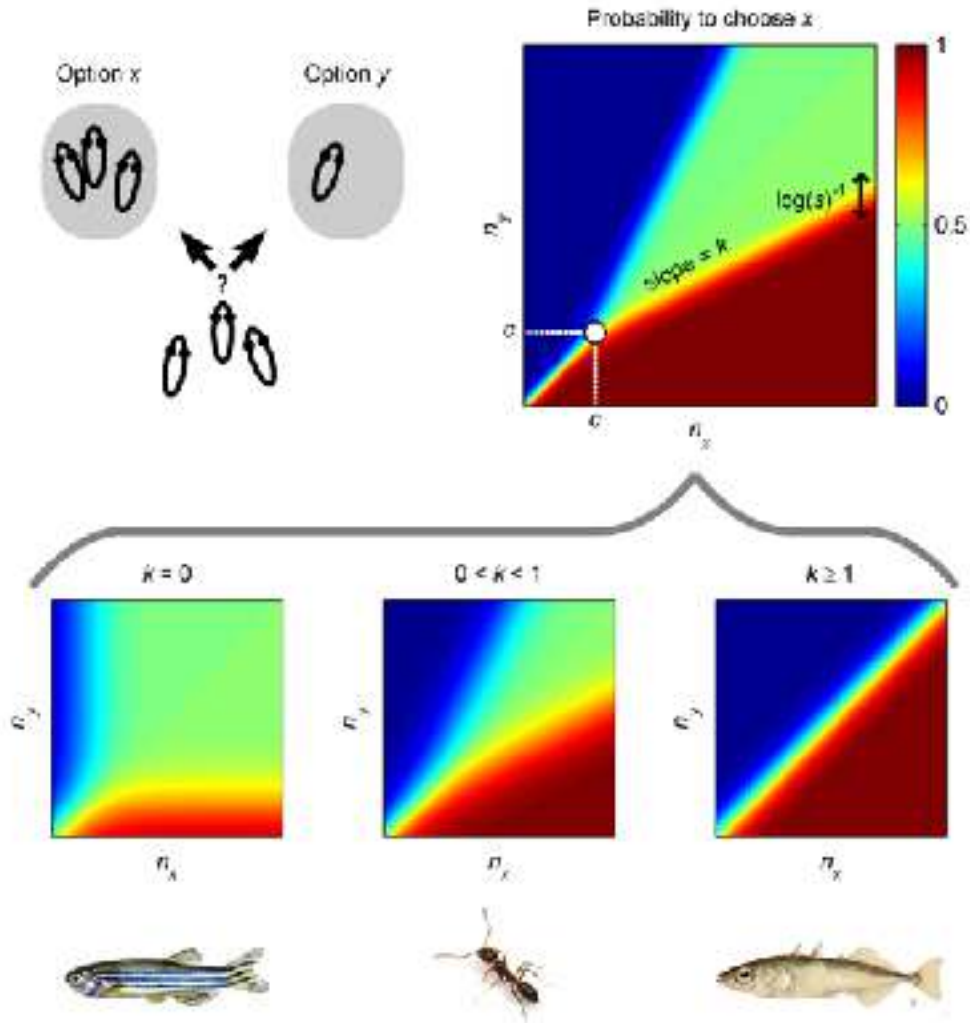
$$P_x = \left( 1 + \frac{1 + aS^{-(n_x - kn_y)}}{1 + aS^{-(n_y - kn_x)}} \right)^{-1}$$



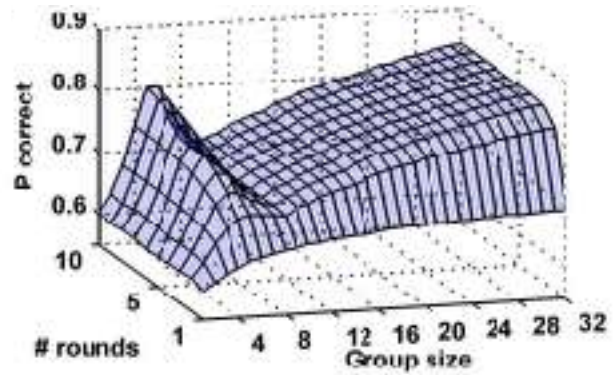
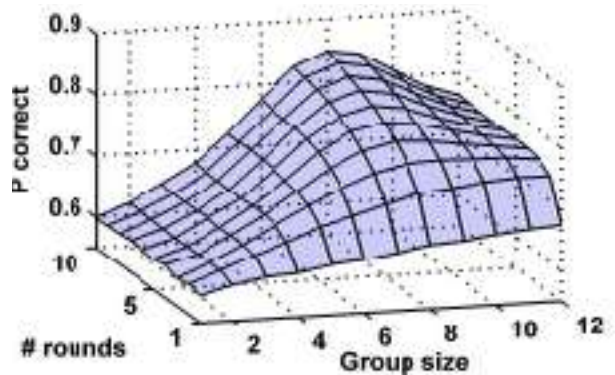
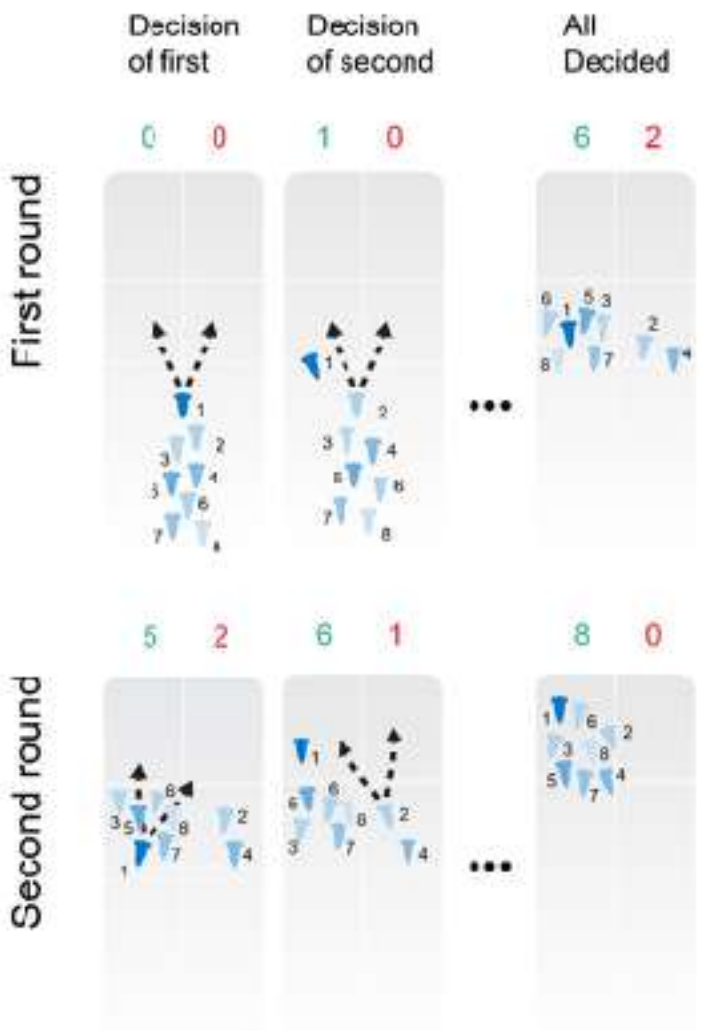
Data from Perna et al. (2012)



Arganda, Perez-Escudero & de Polavieja, *PNAS* (2012)



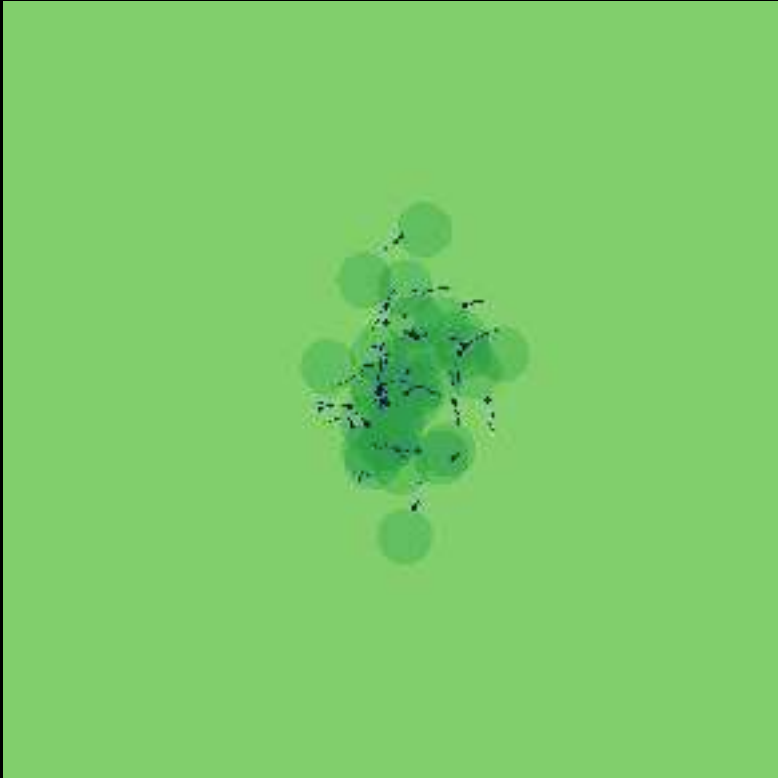
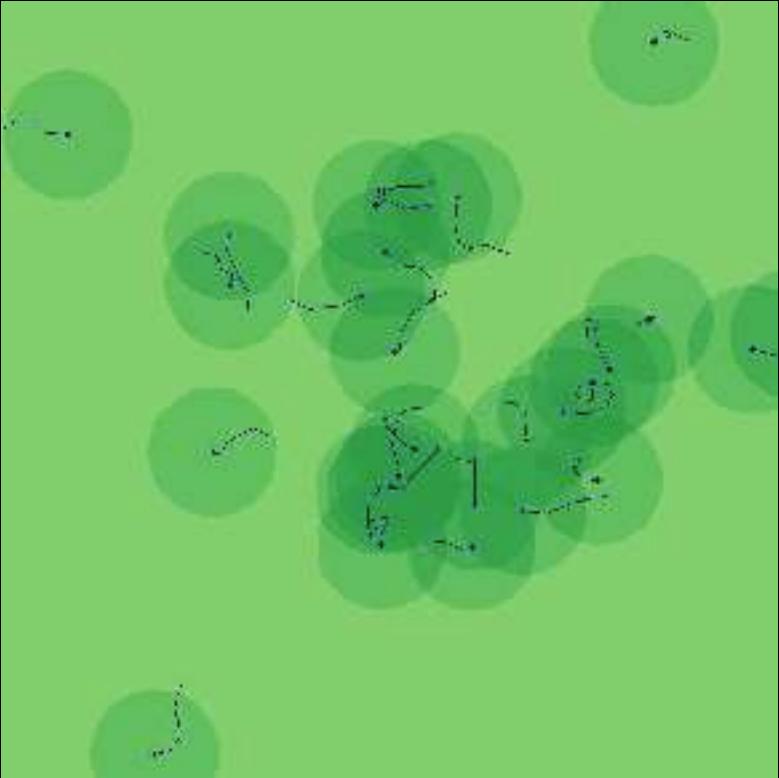
Models allow comparing the real system with virtual alternatives as reference



Vicente, Perez-Escudero & de Polavieja (2017)



Increased aggregation when all options are worse is for free: just changing  $\alpha$



$$P_x = \left( 1 + \frac{1 + aS^{-(n_x - kn_y)}}{1 + aS^{-(n_y - kn_x)}} \right)^{-1}$$

Theory of decision-making in groups



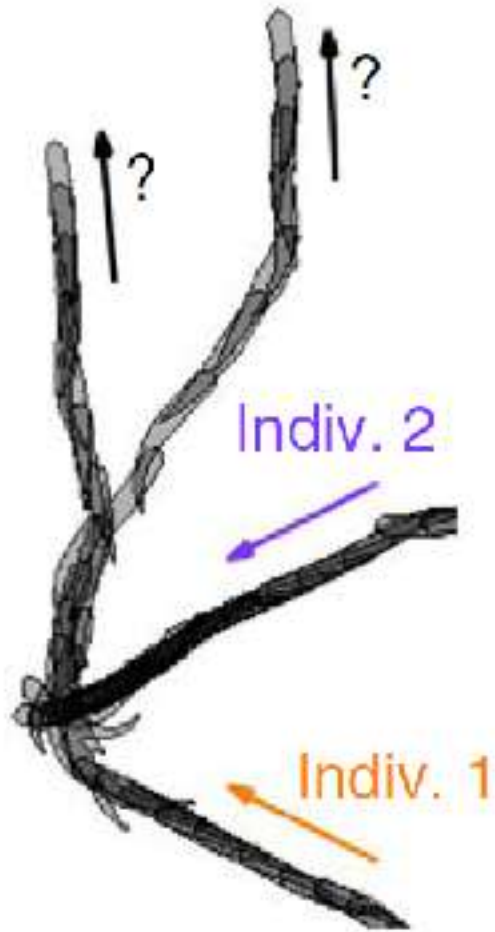
Data-driven study of collective behaviour



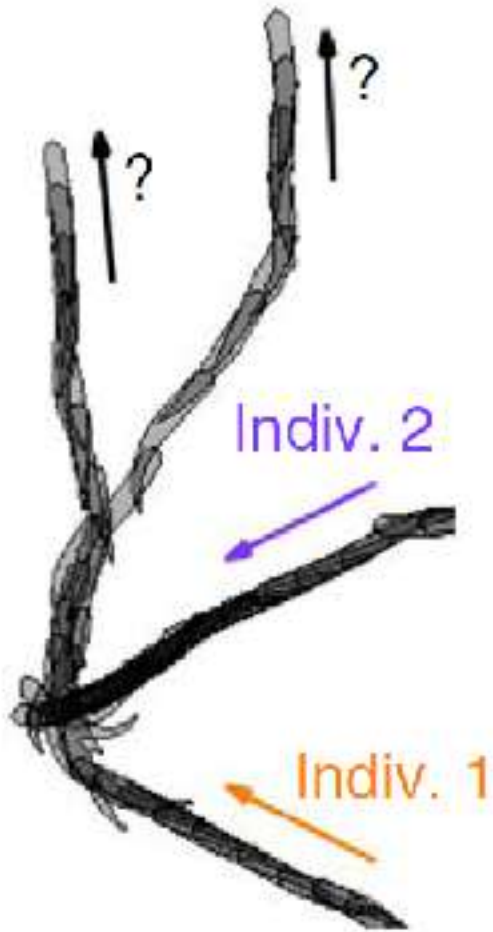
Collective behavior in humans



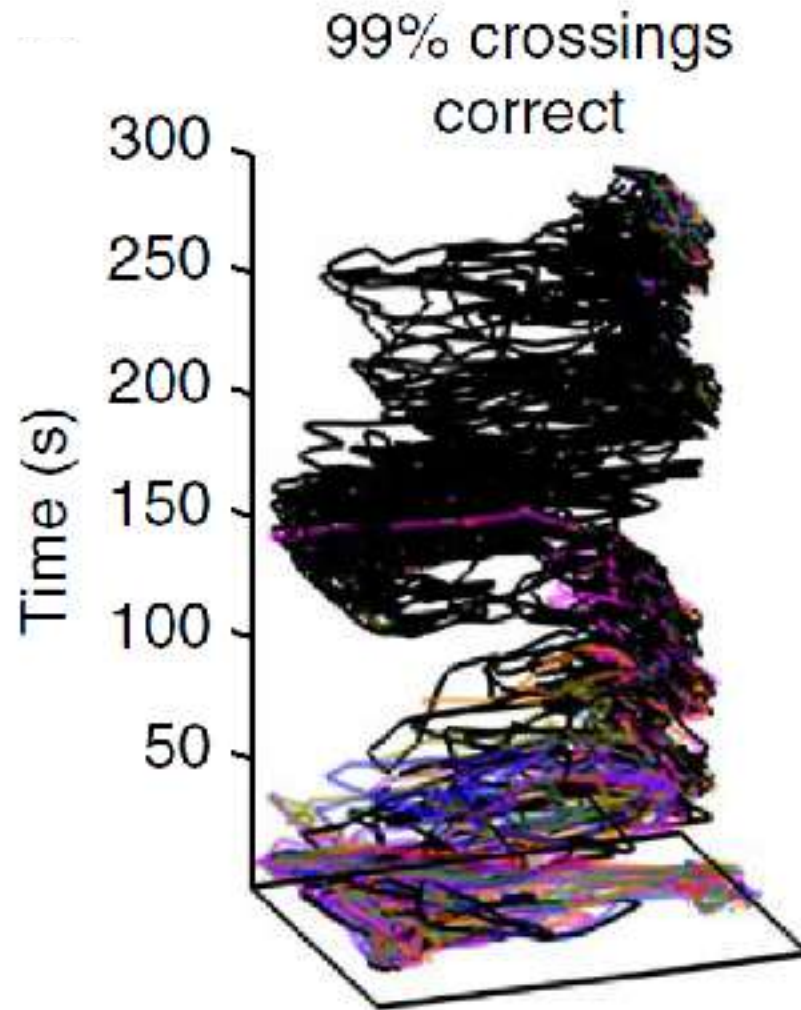
# THE PROBLEM



# THE PROBLEM



# STANDARD SOLUTION



*Danio rerio*

Indiv. 1

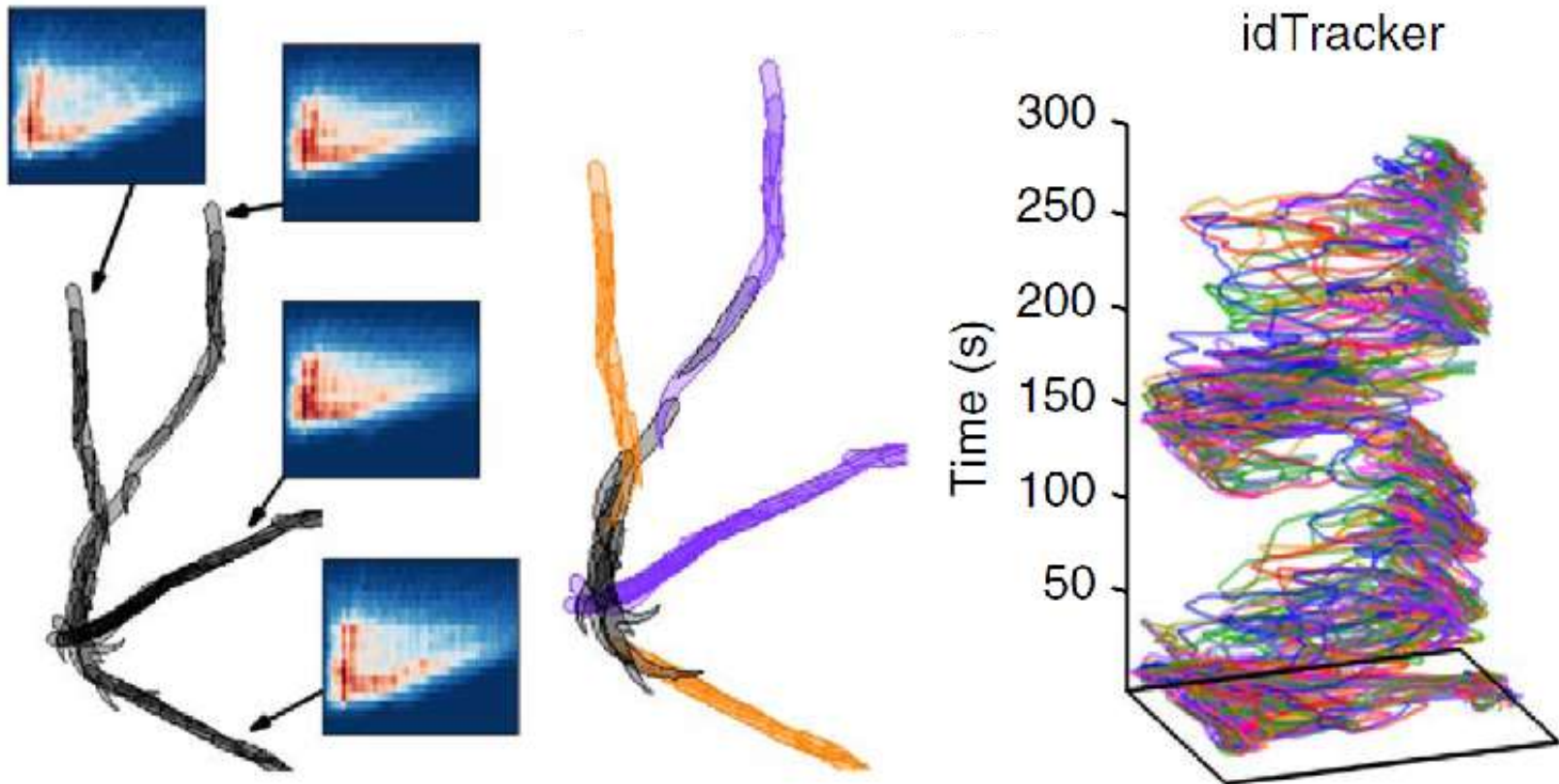
Indiv. 2

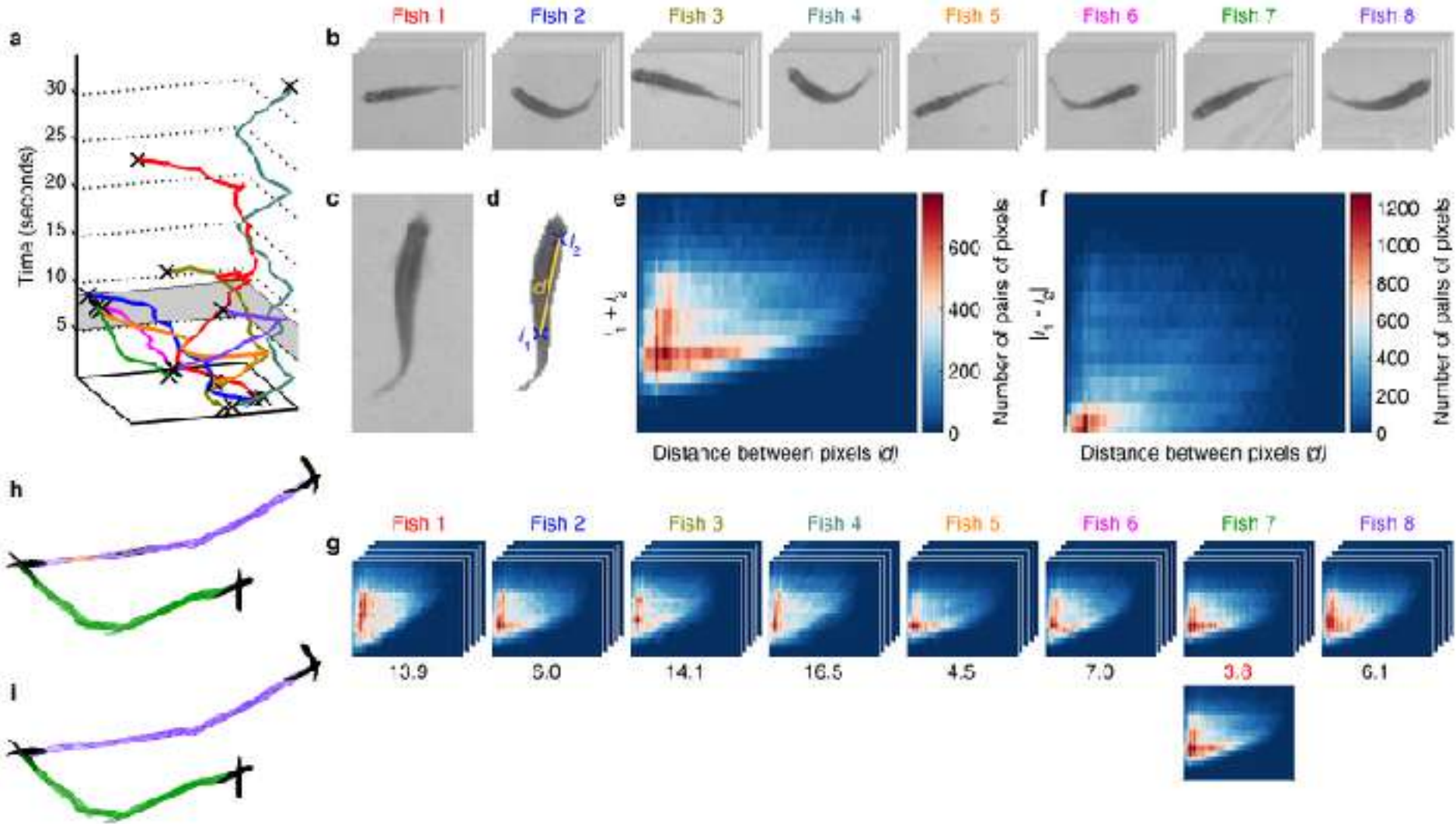
Indiv. 3

Indiv. 4



# TRACKING BY FINGERPRINTING





Handwritten text, possibly a signature or initials, located on the right side of the page.







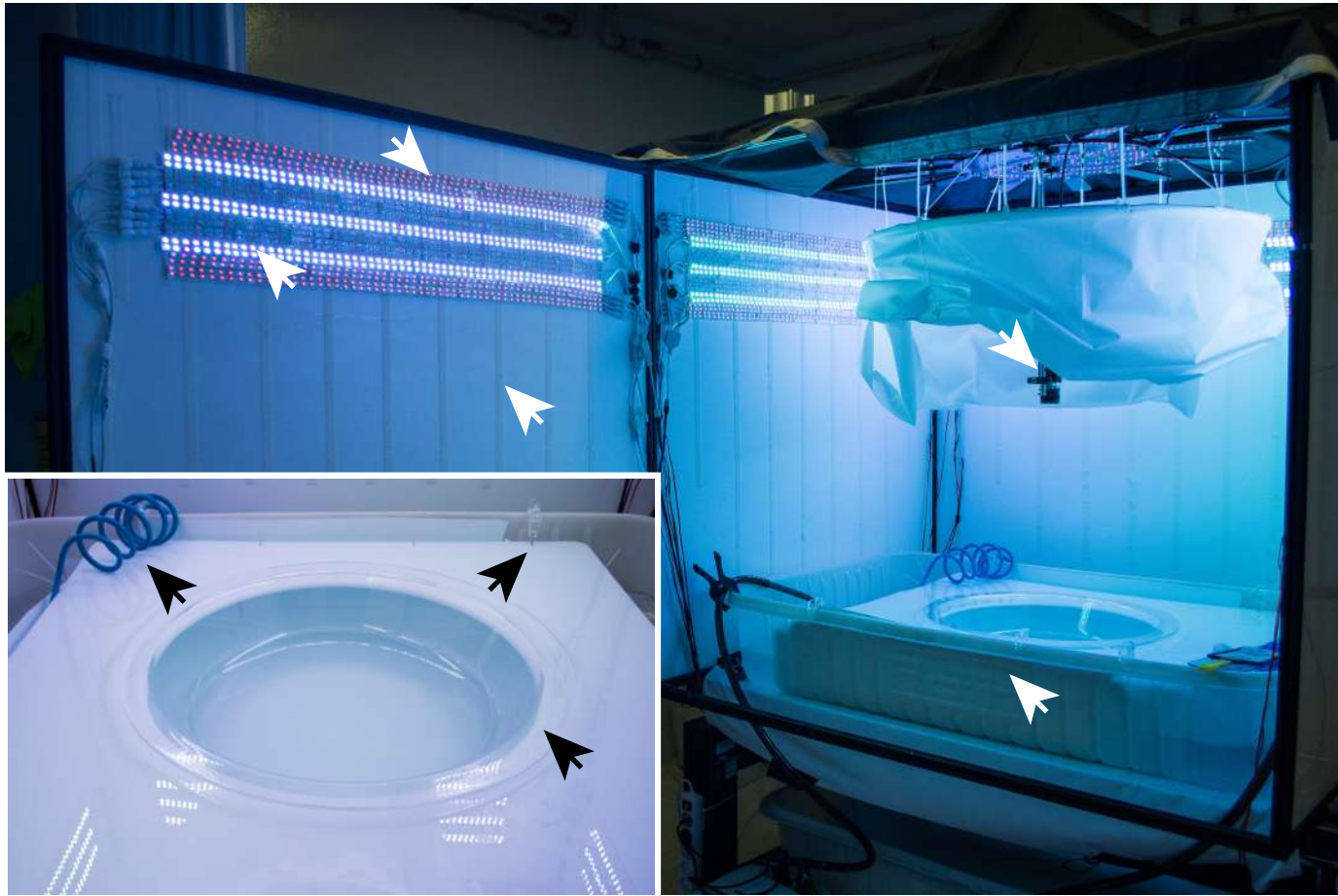
**Accuracy in fish**

**>99.99% accurate**

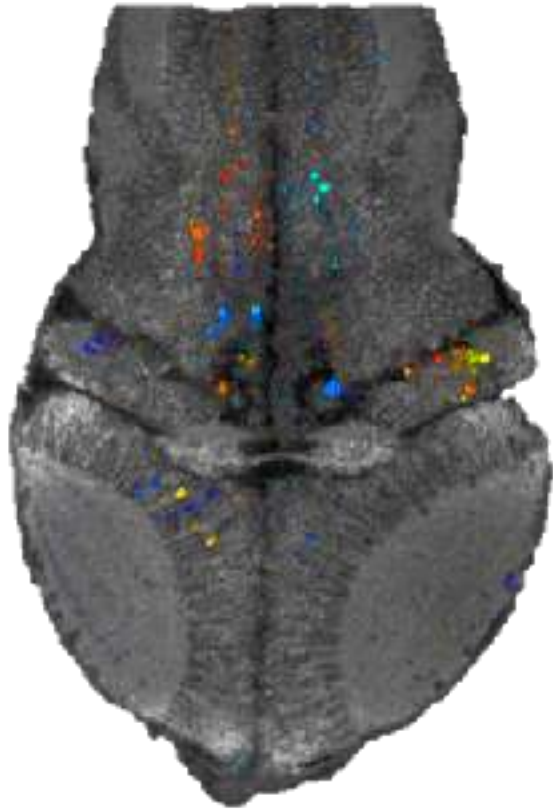
**groups of 60, 80 and 100**

**(still checking in flies)**





# What are our theories based upon?



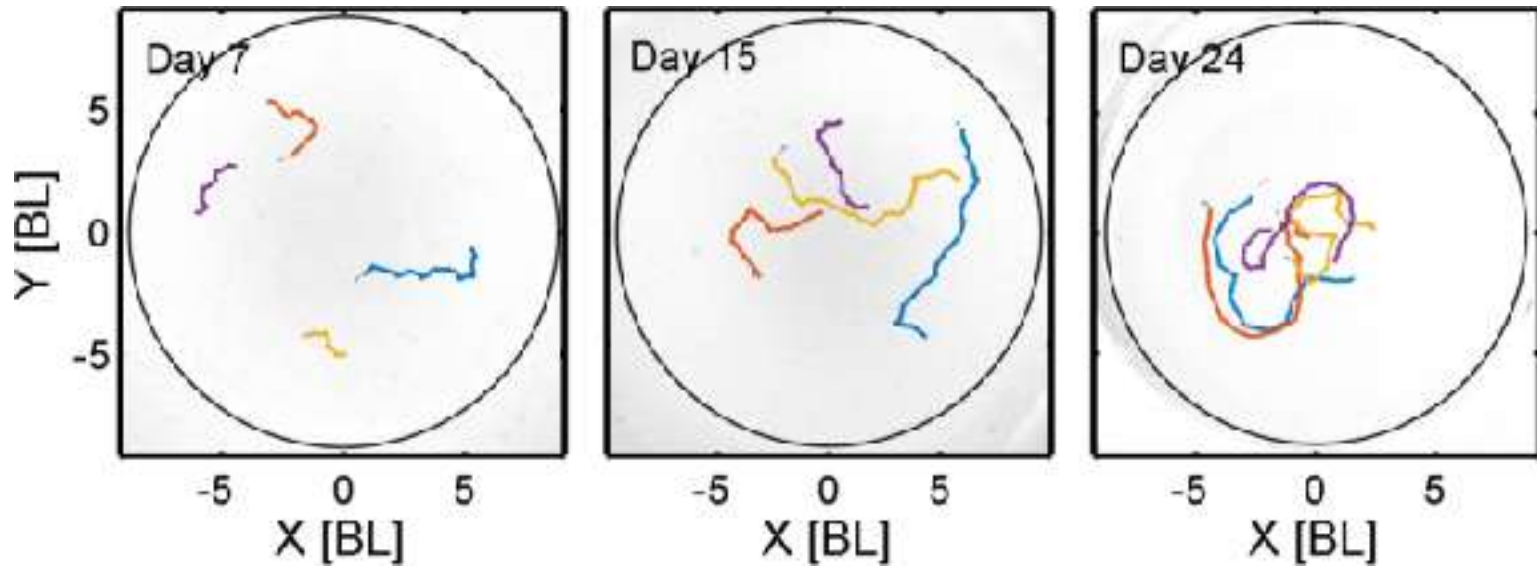
Individuals use **inference**

Individuals learn by **rewards**

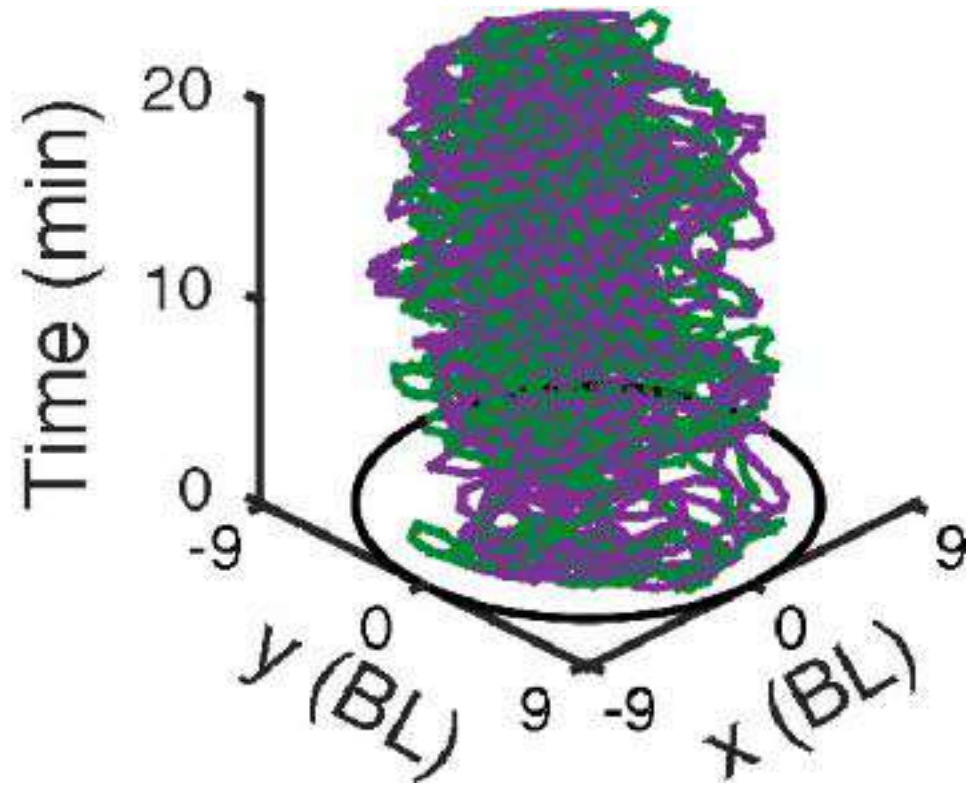
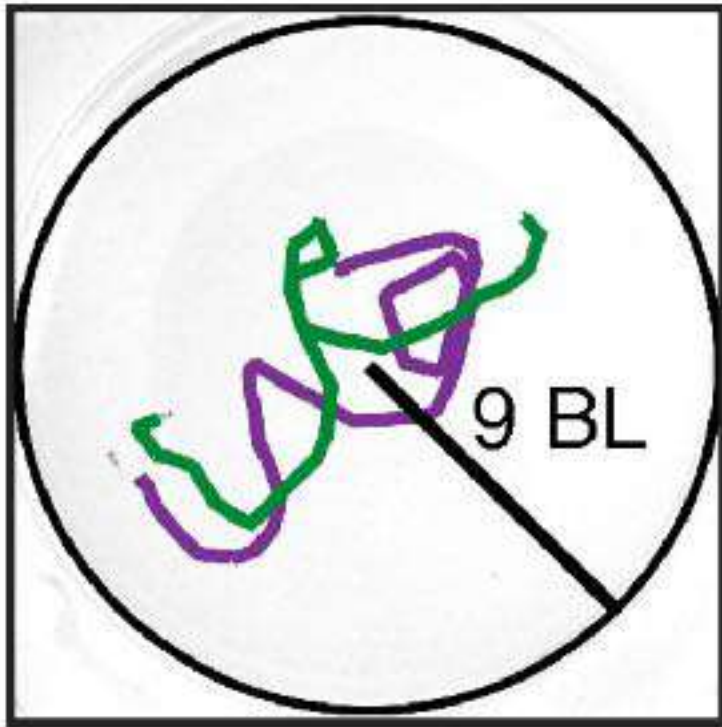
Individuals **control** their behavior according to some policies

Individuals use **heuristic** rules

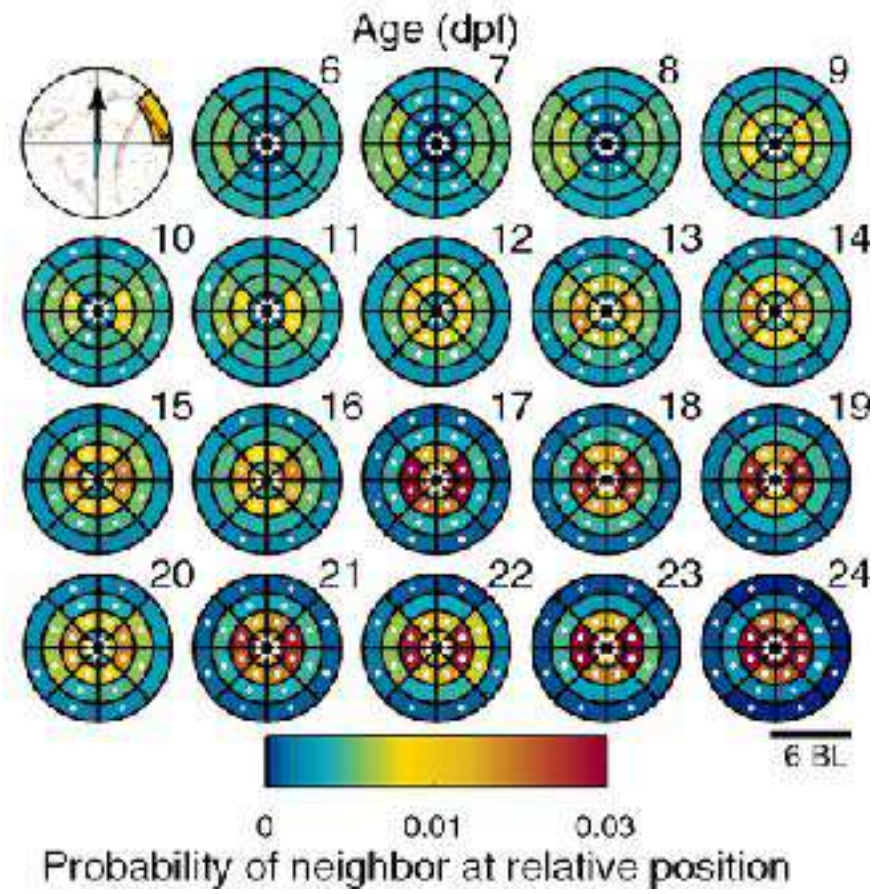
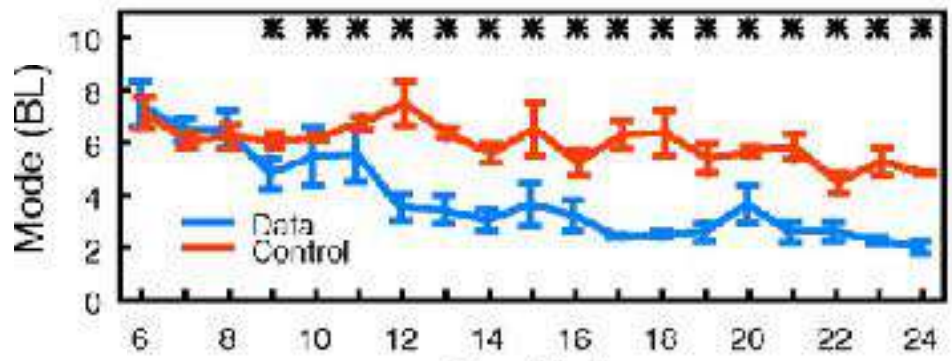
Individuals are a **neuronal network** coupled to a skeletal system

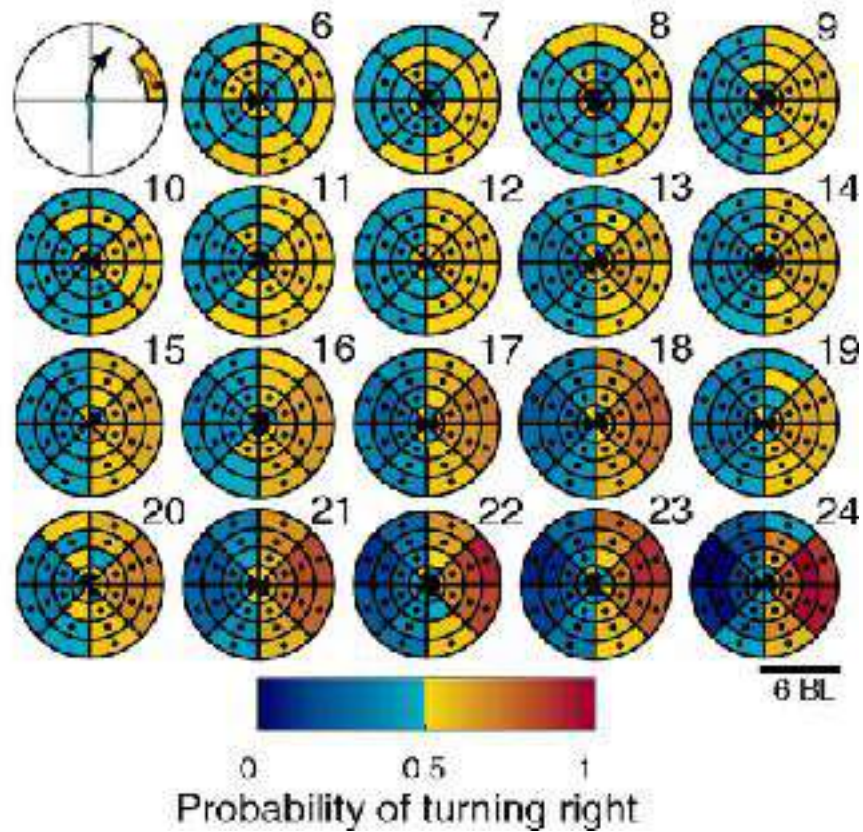
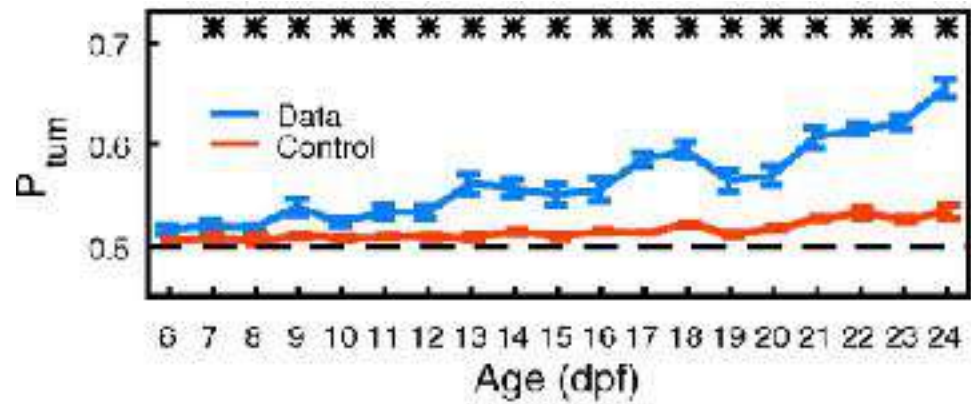


500 videos during 6-24 dpf

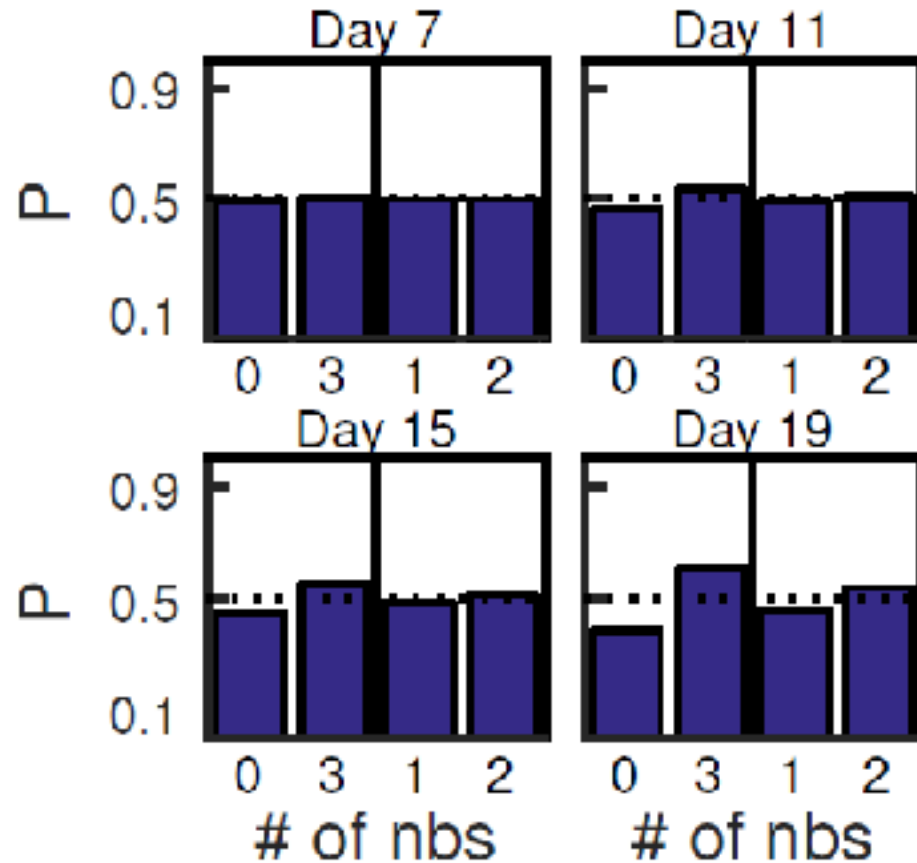




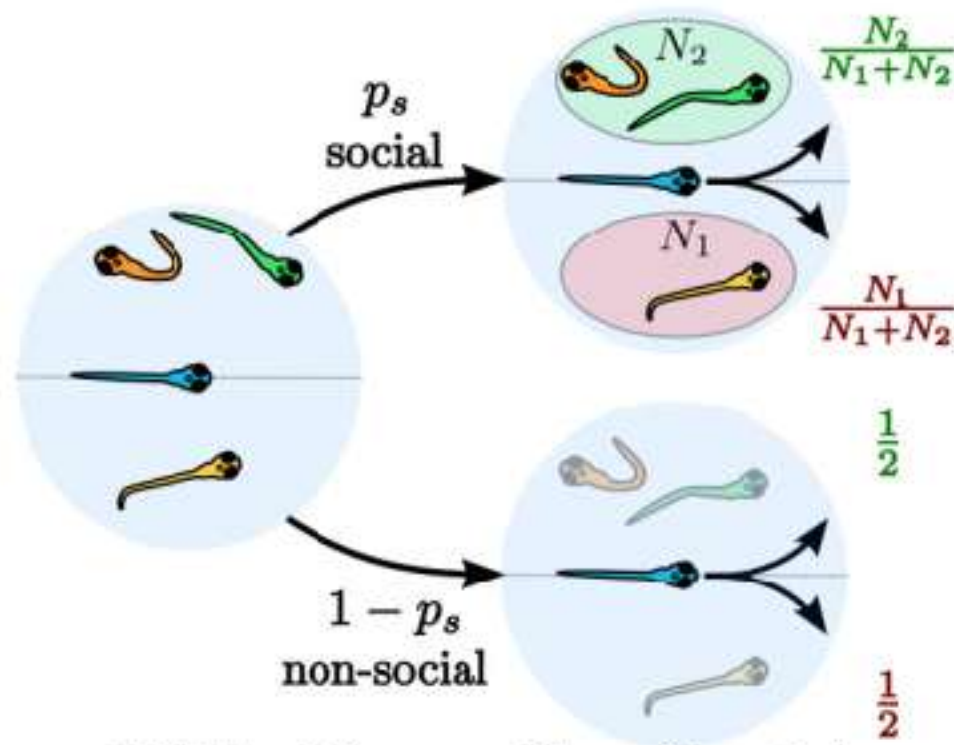




# Experiments with 4 fish



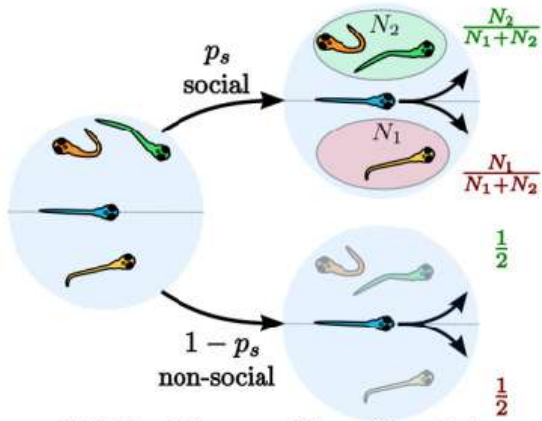
# Moving towards an agent chosen at random



$$P(N_1 | N_1 : N_2) = p_s \frac{N_1}{N_1 + N_2} + (1 - p_s) \frac{1}{2}$$



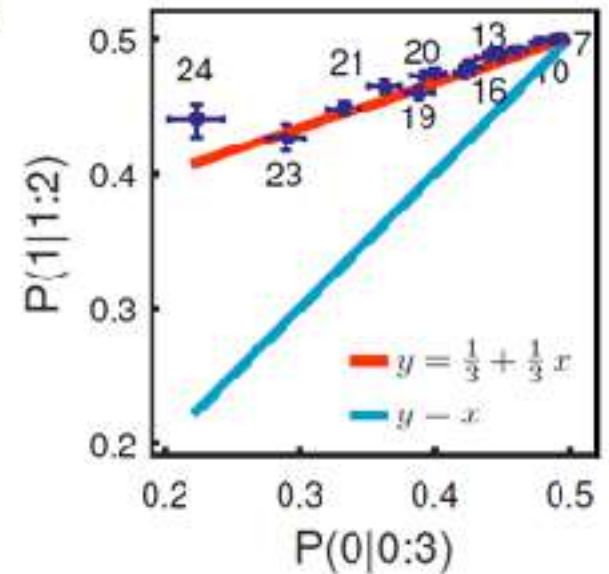
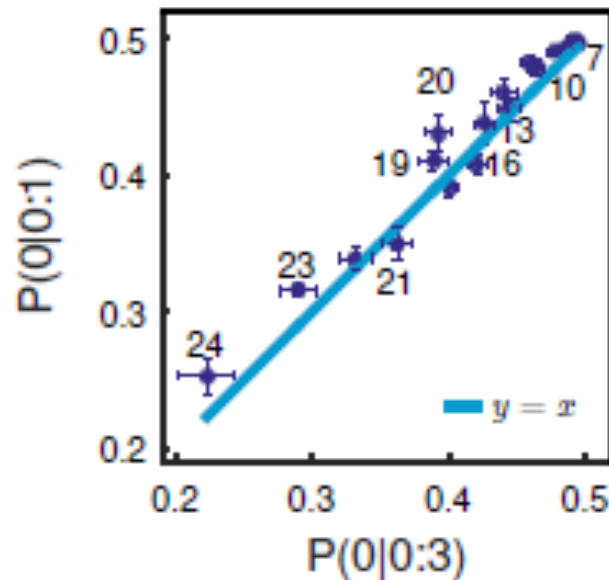
# Parameter-free predictions of model



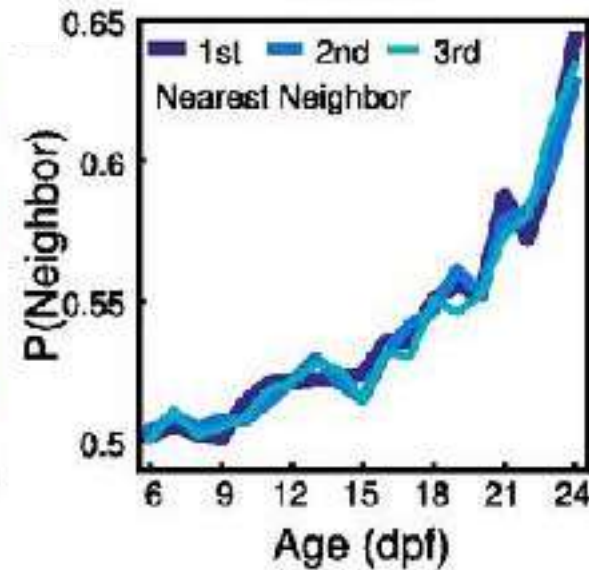
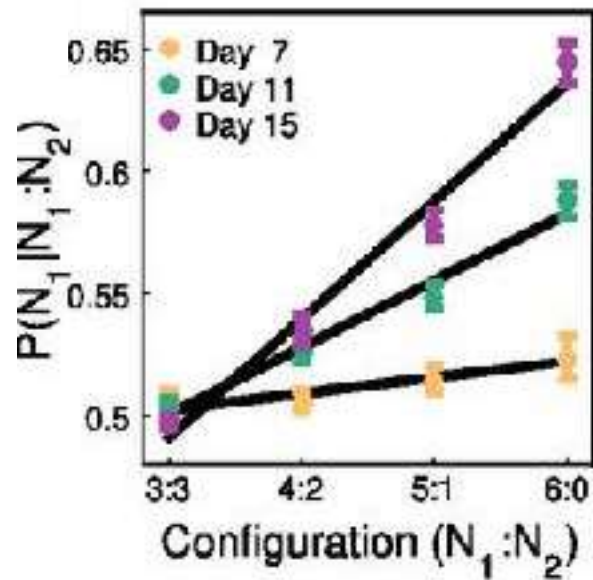
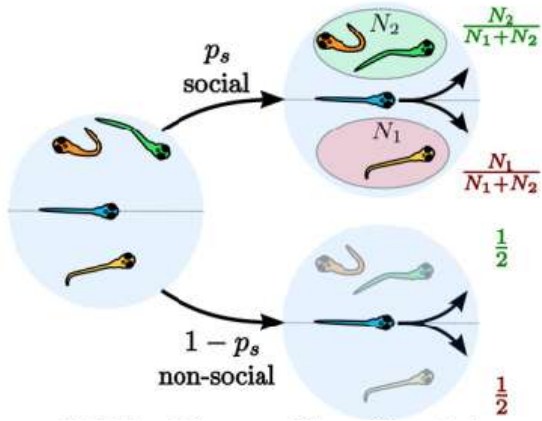
$$P(N_1|N_1 : N_2) = \frac{1}{2} + \frac{(N_1 - N_2)(\bar{N}_1 - \bar{N}_2)}{(N_1 + N_2)(\bar{N}_1 - \bar{N}_2)} \left( P(N_1|N_1 : N_2) - \frac{1}{2} \right)$$

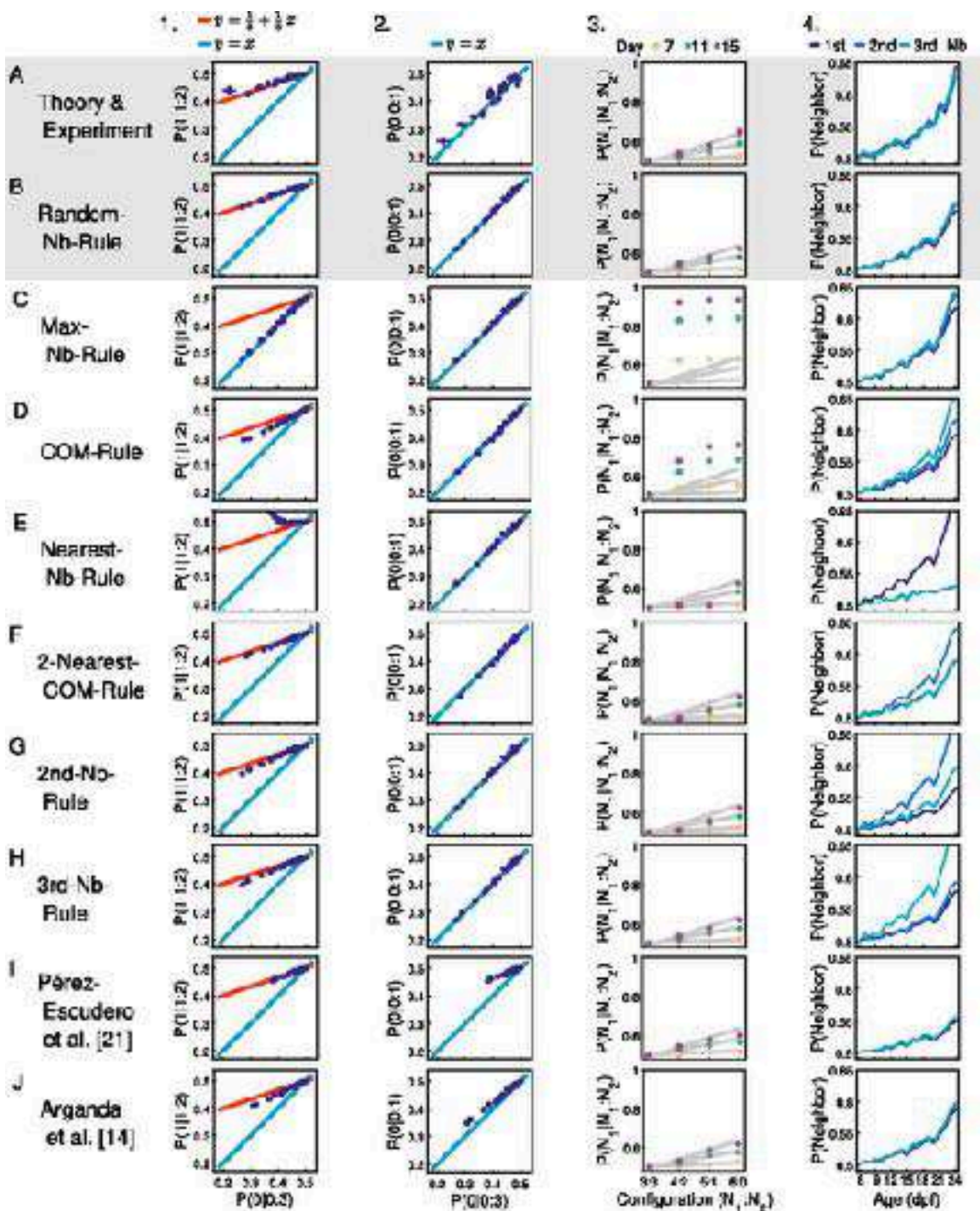
$$P(0|0 : 1) = P(0|0 : 3)$$

$$P(1|1 : 2) = \frac{1}{3} + \frac{1}{3}P(0|0 : 3)$$



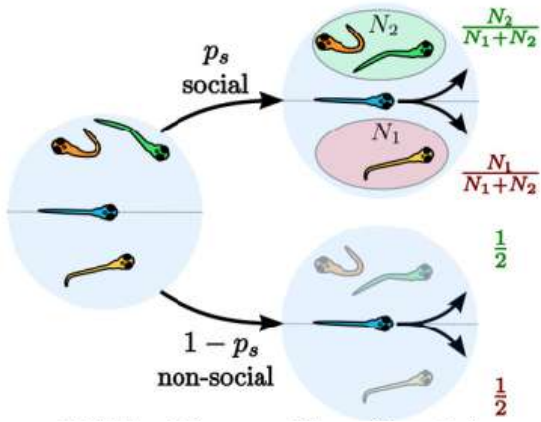
# Other predictions



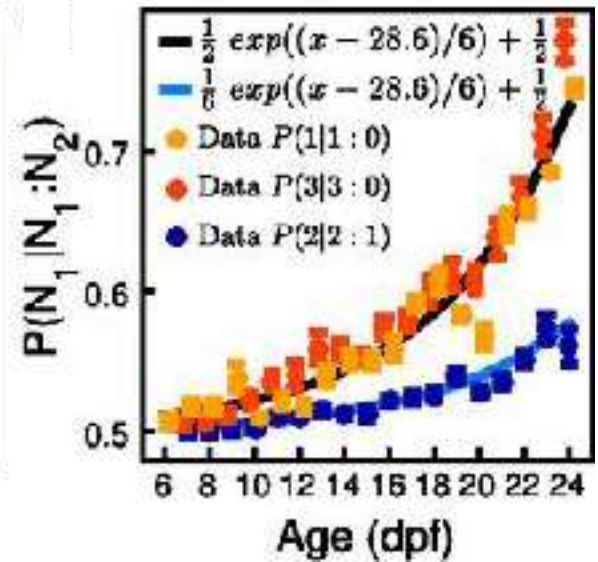
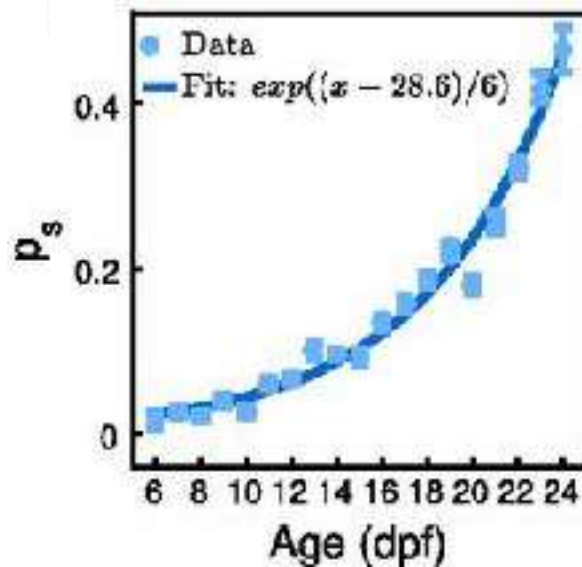


comparison with other models

# Changes during development

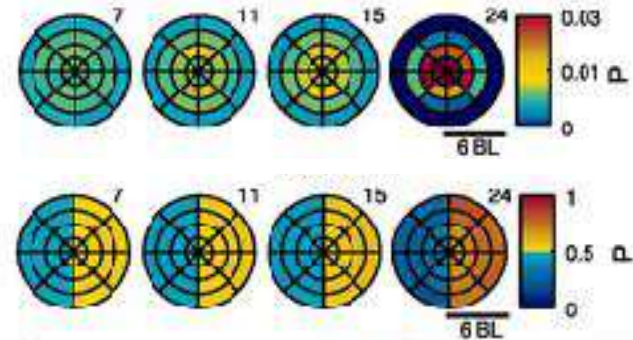
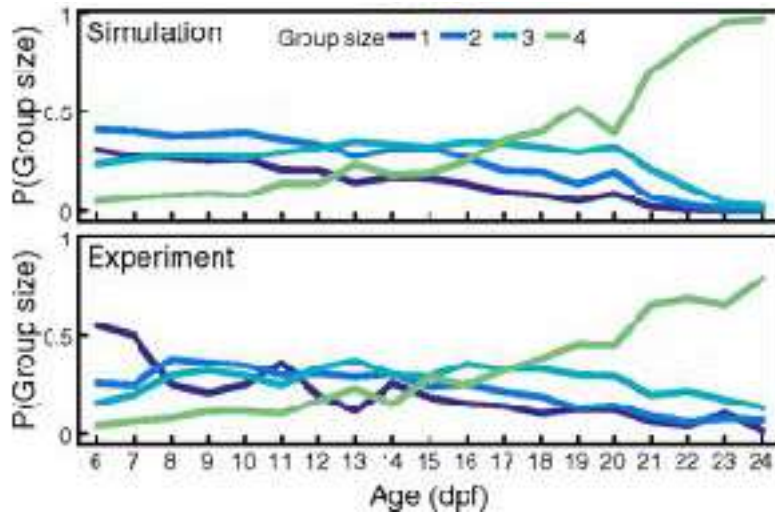
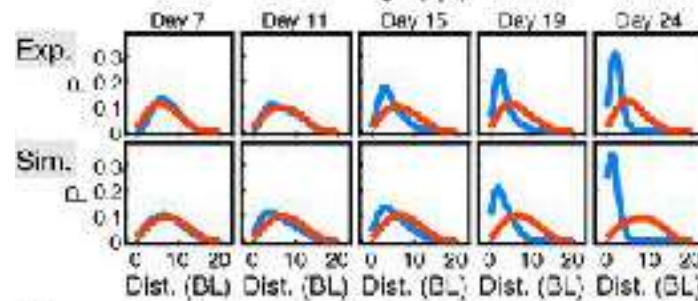
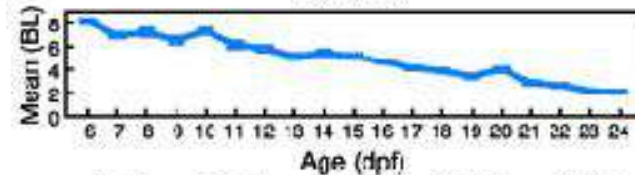
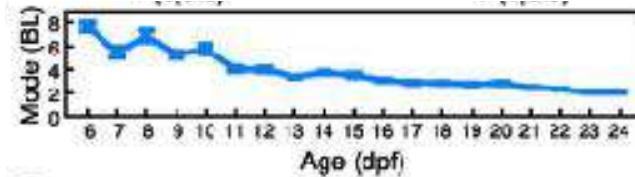
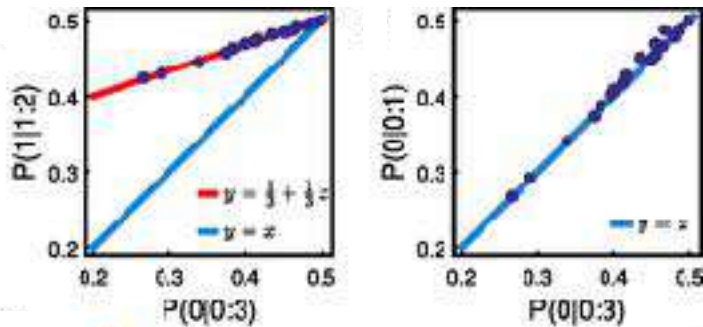


$$P(N_1|N_1 : N_2) = p_s \frac{N_1}{N_1 + N_2} + (1 - p_s) \frac{1}{2}$$





# Running the model & analyse it as exp



$$P_x = \left( 1 + \frac{1 + aS^{-(n_x - kn_y)}}{1 + aS^{-(n_y - kn_x)}} \right)^{-1}$$

Theory of decision-making in groups

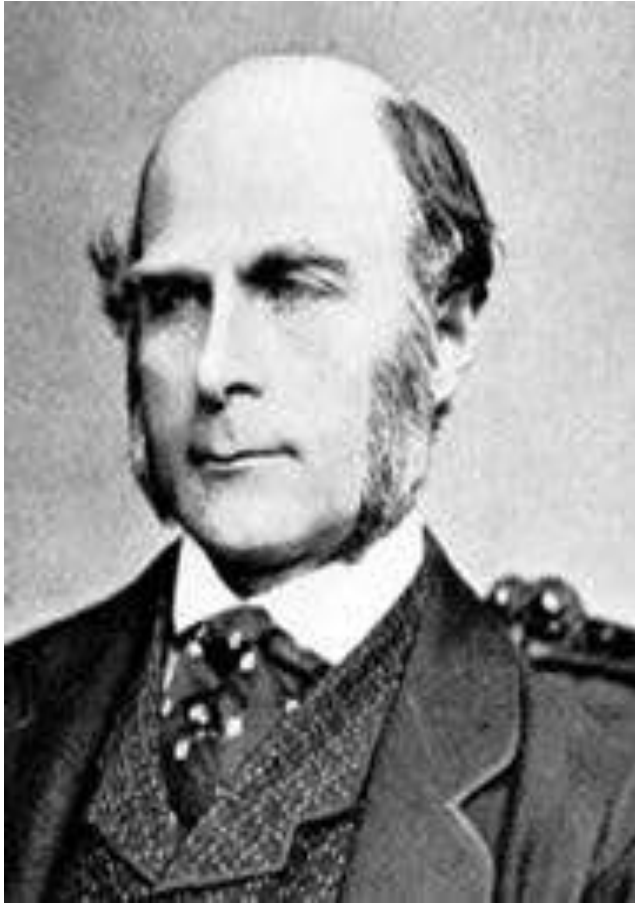


Data-driven study of collective behaviour



Collective behavior in humans

# Galton (1907)



## Francis Galton

### *Vox Populi*

IN these democratic days, any investigation into the trustworthiness and peculiarities of popular judgments is of interest. The material about to be discussed refers to a small matter, but is much to the point.

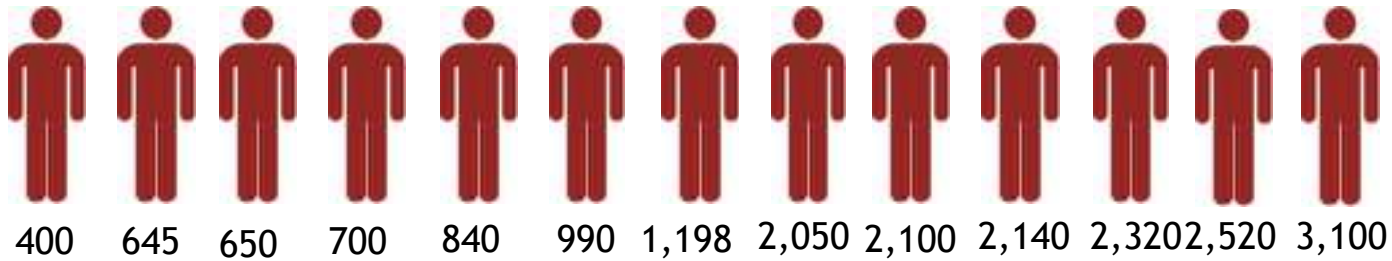


# Galton (1907)

What is the weight of the ox?



Individuals (800) write down their answer independently of each other



Median value (middle observation of ordered list) = 1,198  
Real value = 1,207 } 1 % error

What is the border length between Italy and Switzerland?

Median value = 302  
Real value = 734 } 60 % error

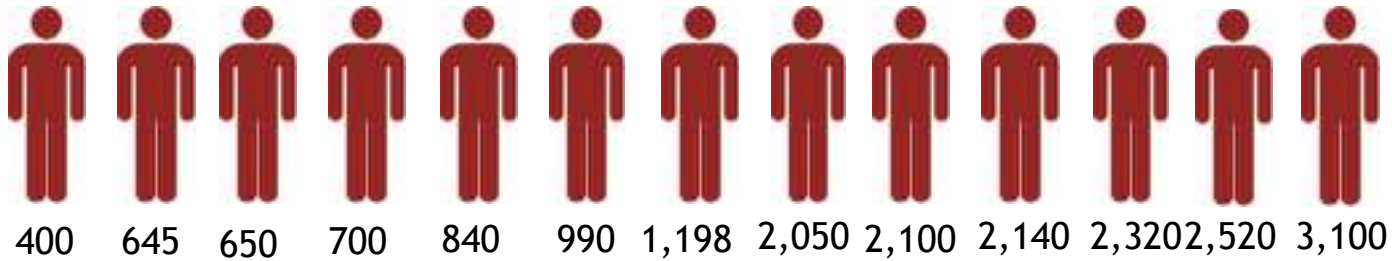


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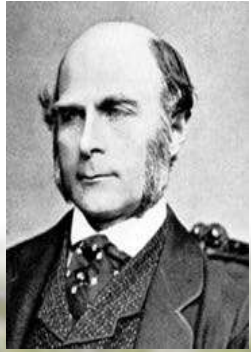


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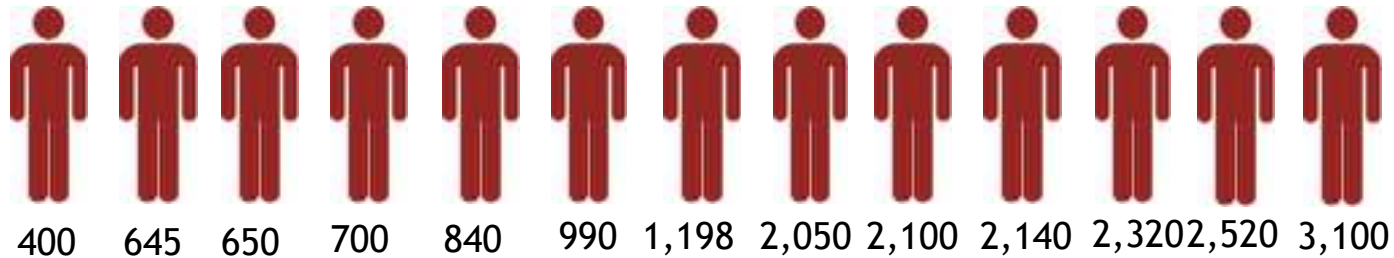
What is the weight of this ox?



# Wisdom of the crowd

What is the weight of the ox?

Individuals (800) write down their answer independently of each other



Median value (middle observation of ordered list) = 1,198  
Real value = 1,207 } 1 % error

What is the border length between Italy and Switzerland? From Lorenz et al (2011)

Median value = 302  
Real value = 734 } 60 % error

# Simple interactions make matters worse

What is the border length between Italy and Switzerland?

Median value  $302 \pm 495$   
Real value = 734 } 60 % error

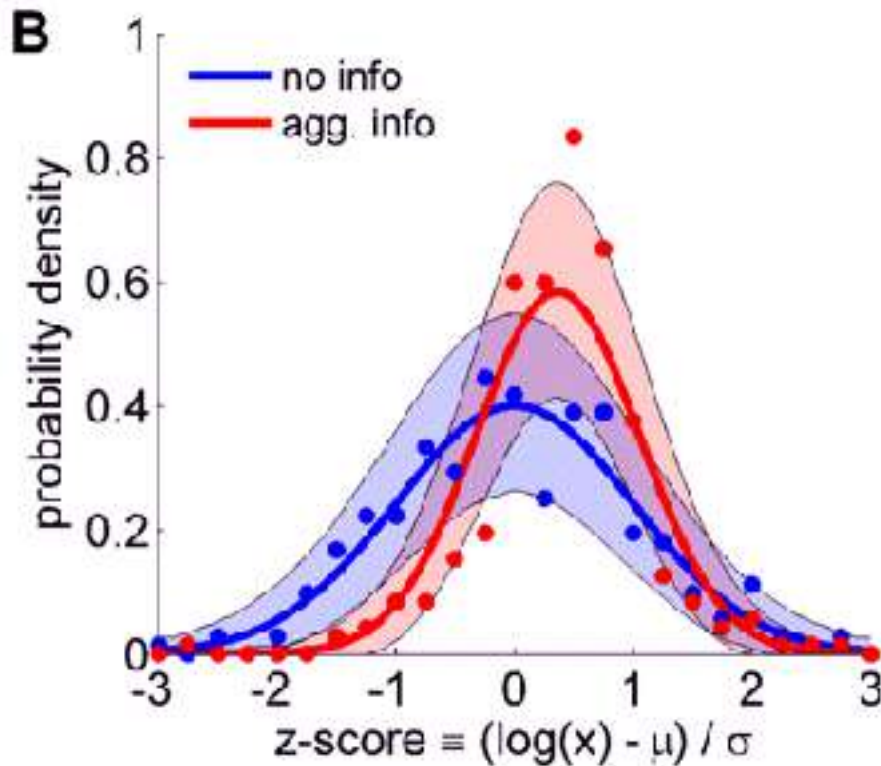
Give all people the mean value, and ask again

What is the border length between Italy and Switzerland?

Median value =  $381 \pm 279$  They just copied

Using again Bayes Theorem and that counting distributions tend to be log-normal, we obtain for  $y = \log(x)$

$$P(y|p, s) \propto P(y|s)P(s|p, y) = \mathcal{N}(\mu_f, \sigma_f)$$



$$\mu_f = (1 - w_s)\mu_p + w_s\mu_s$$

$$\sigma_f = \sqrt{1 - w_s}\sigma_p$$

A model for an individual compatible with the distributions is

$$x_2 = x_1^{1-w_s} \bar{x}_1^{w_s}$$

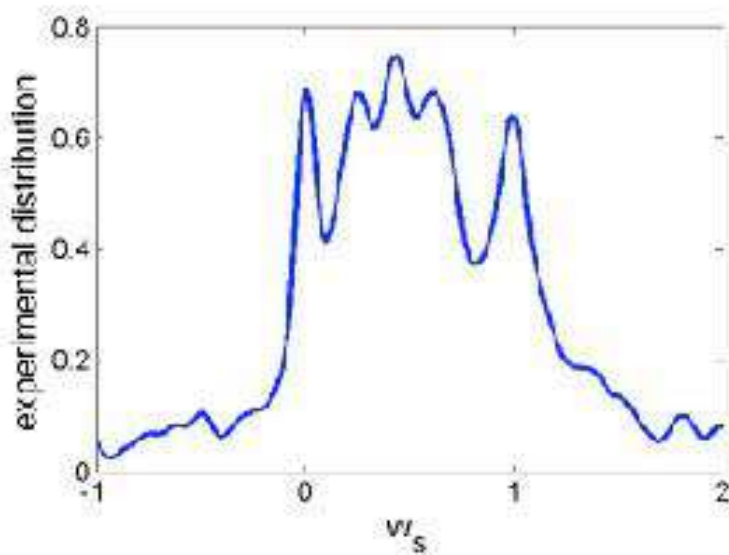
Social weight
Social value
First (private) estimation

Second estimation

So from the two estimations, for each individual we can extract a social weight as

$$w_s = \frac{\log(x_2) - \log(x_1)}{\log(\bar{x}_1) - \log(x_1)}$$

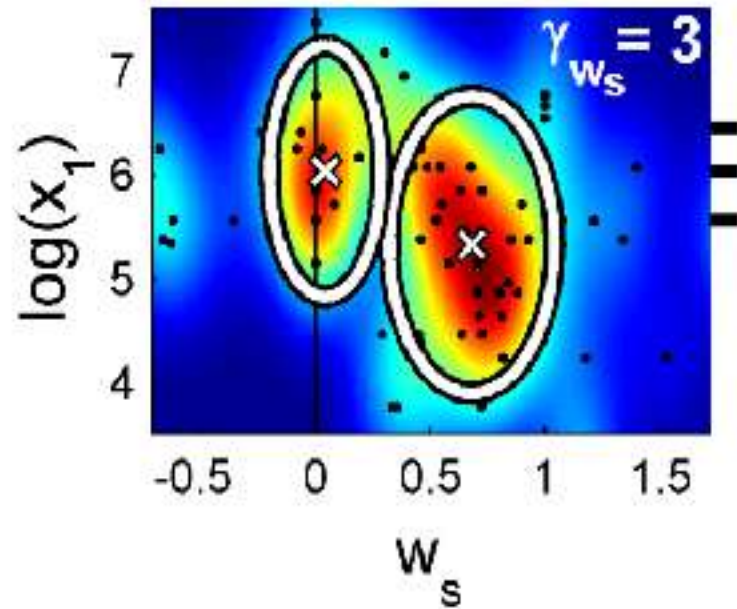
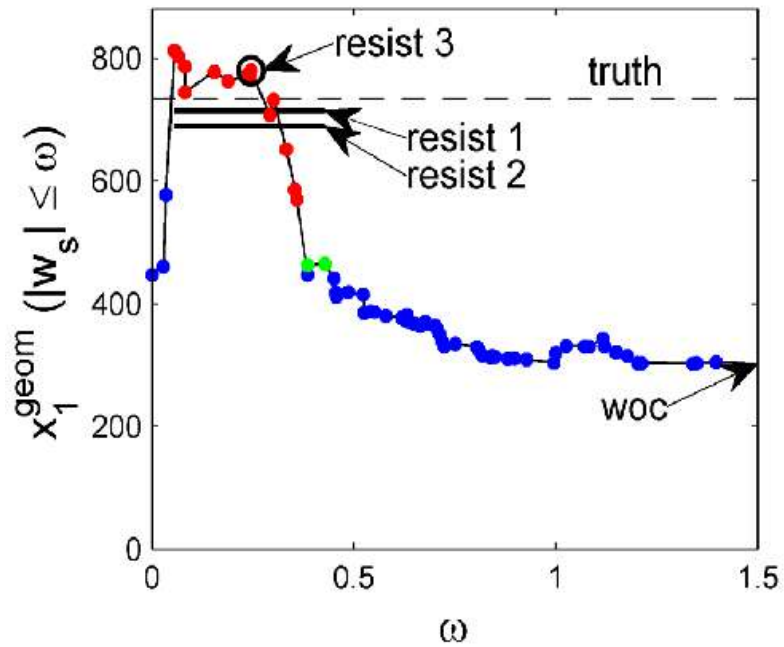
Change in opinion
Distance from first estimation to social value



**HYPOTHESIS:**

Resistance to social influence statistically correlates with accuracy



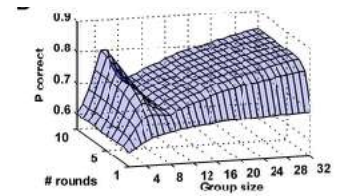
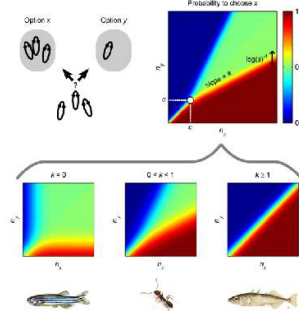
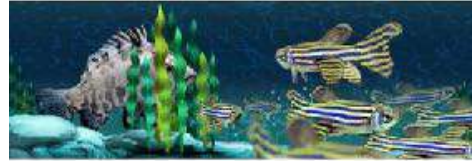


<b>Question</b>	<b>Truth</b>	<b>WOC</b>	<b>Resist</b>
<b>Border Length</b>	734	302 (-59%)	715 (-2.6%)
<b>Rapes</b>	639	257 (-60%)	624 (-2.4%)
<b>Assaults</b>	9272	3685 (-60%)	7721 (-17%)
<b>Population Density</b>	184	115 (-38%)	168 (-8.9%)

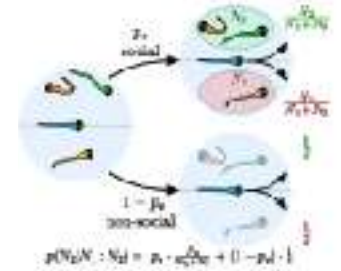
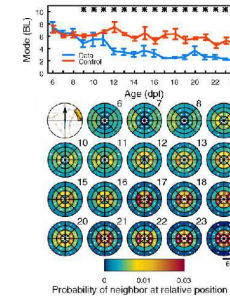
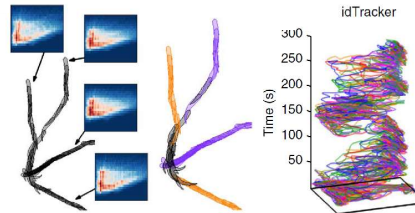


# Theory of decision-making in groups

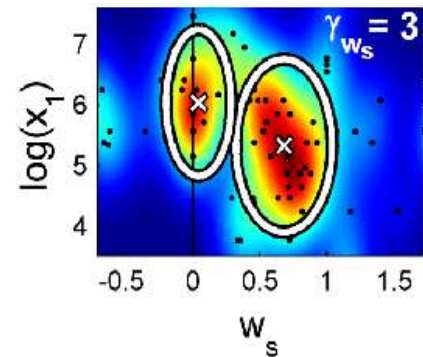
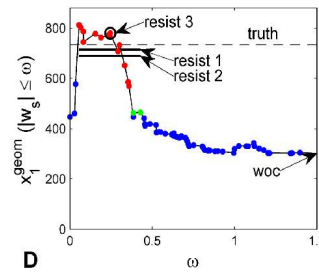
$$P_x = \left( 1 + \frac{1 + aS^{-(n_x - kn_y)}}{1 + aS^{-(n_y - kn_x)}} \right)^{-1}$$



# Data-driven study of collective behaviour



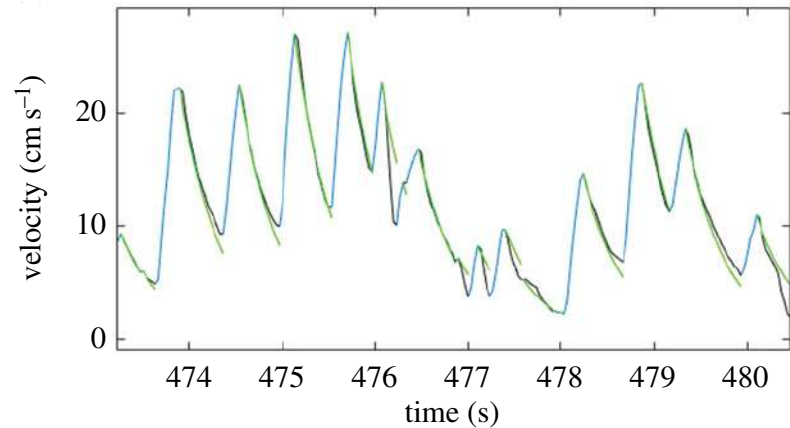
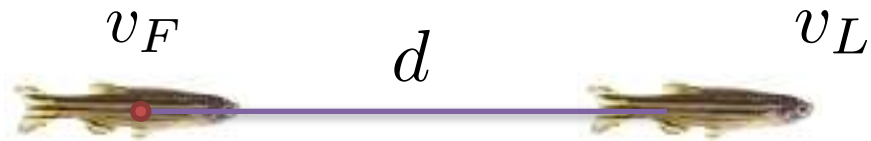
# Collective behavior in humans







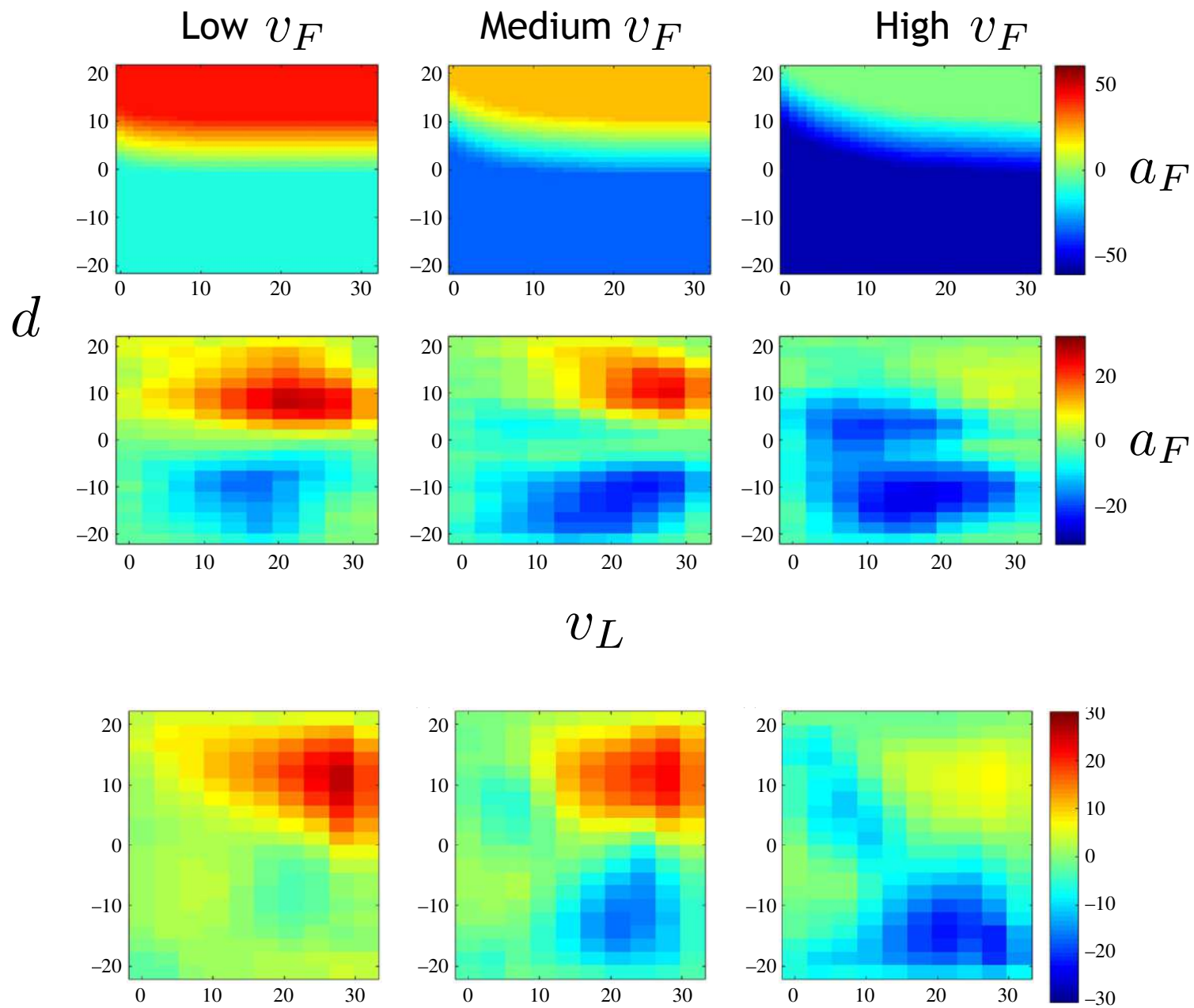
# How do two fish get close? The minimum time solution



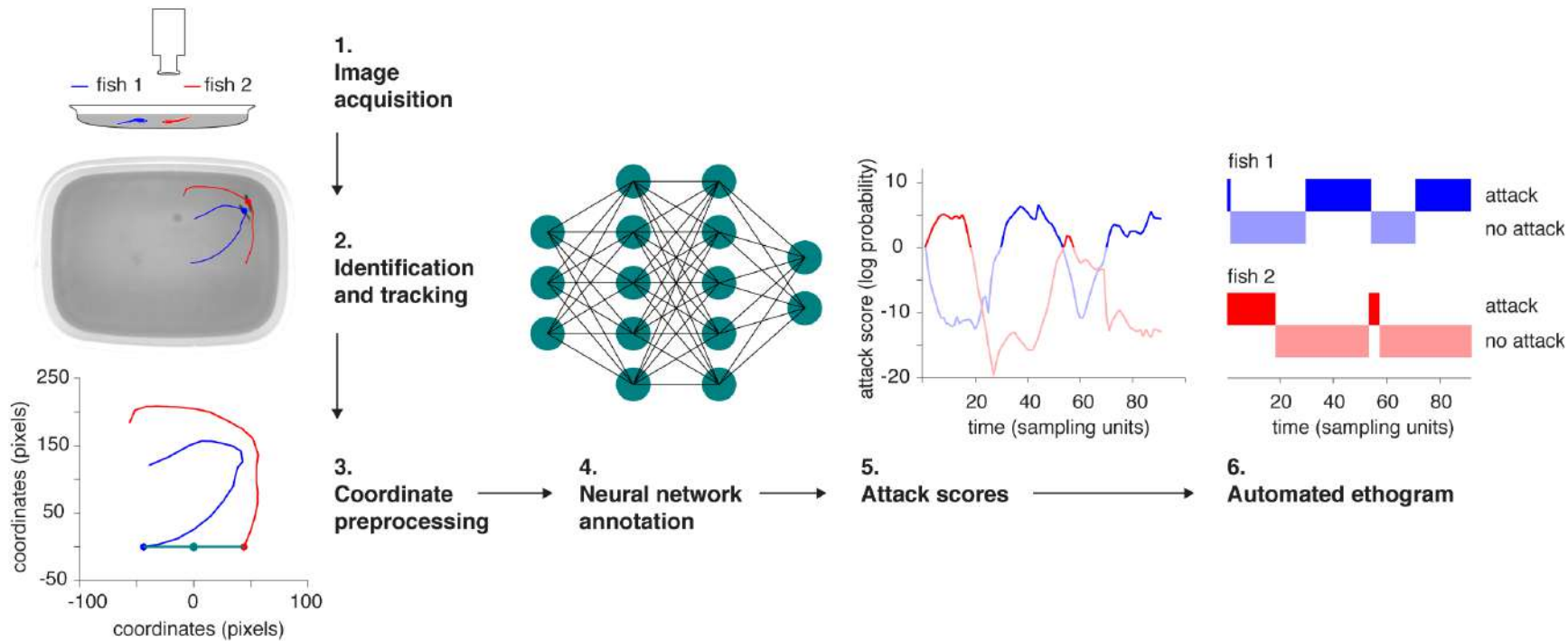
$$v_F(t) = v_F(0) \exp(-\alpha t)$$

$$\alpha = 2.7 \text{ s}^{-1}$$

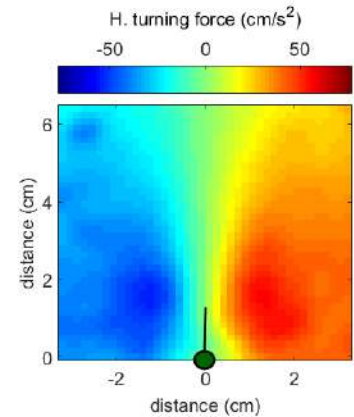
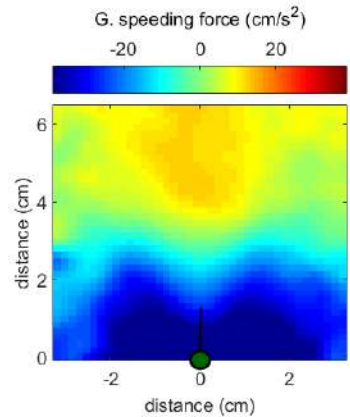
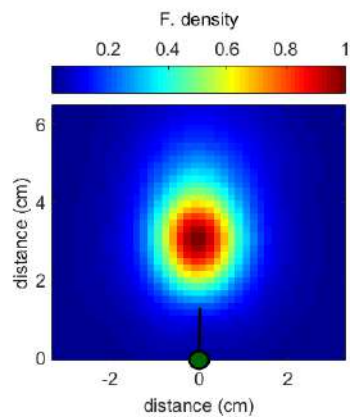
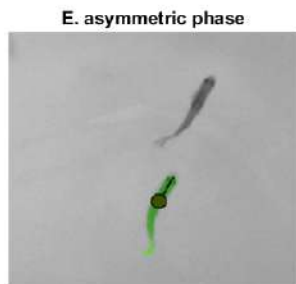
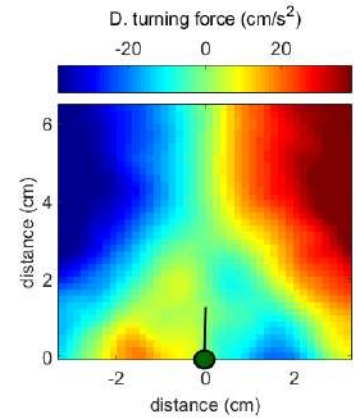
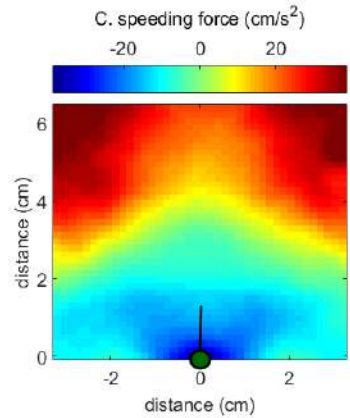
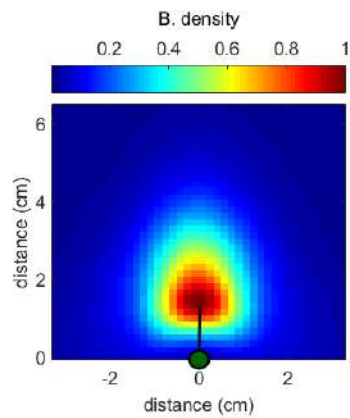
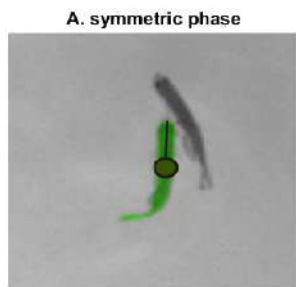
$$\frac{1}{2.7} (v_F - v_L) = d + \frac{v_L}{2.7} \log \frac{v_F}{v_L}$$



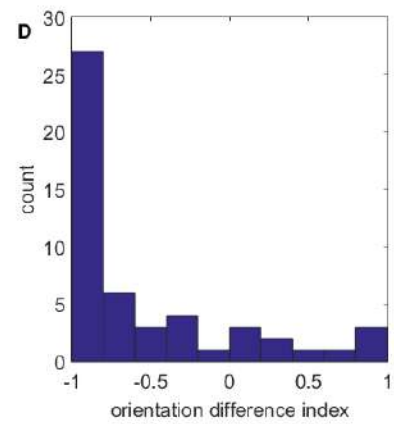
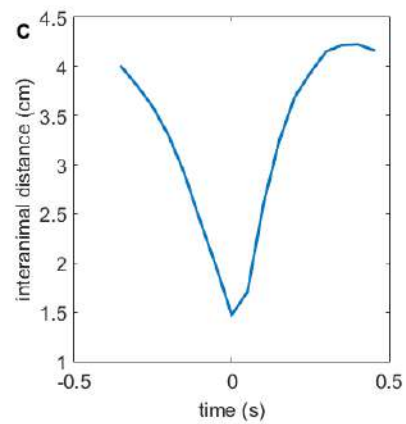
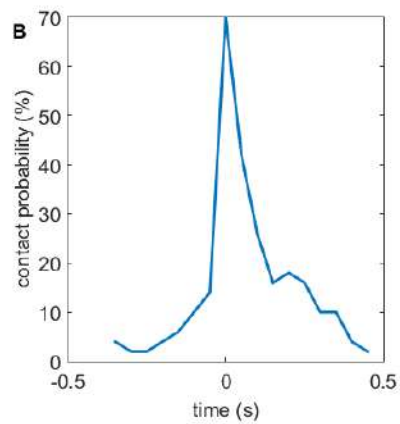
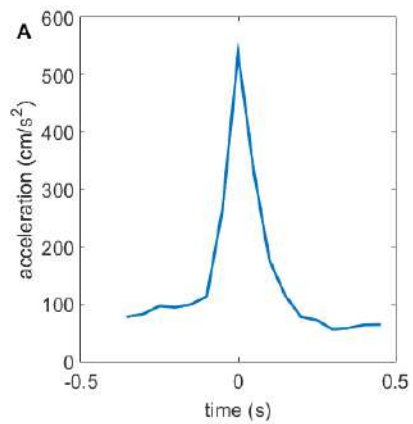
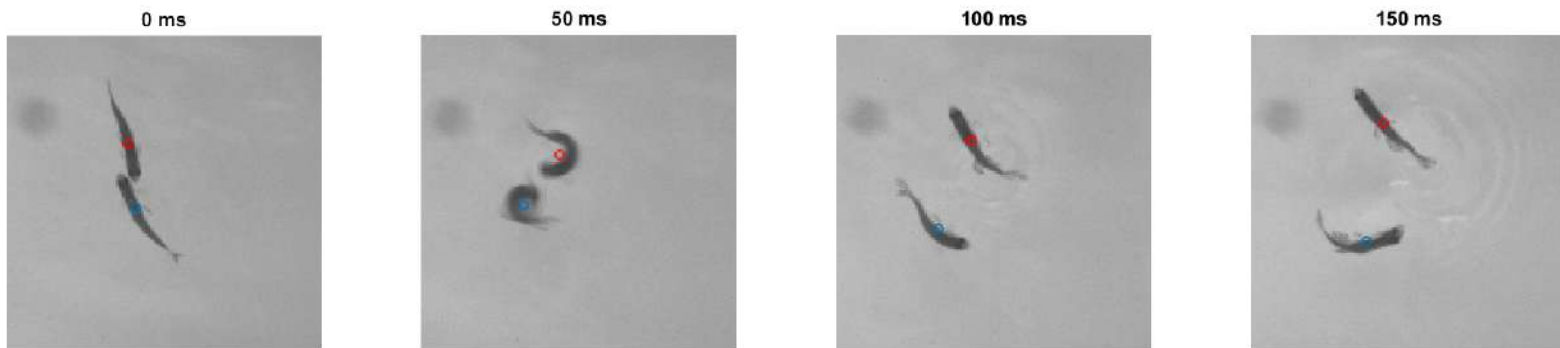













# Do you like this video?

Video 1/7

Click on the image below to watch the video and to see how other users evaluate it.



For each of the statements below please let us know how much you agree or disagree with the statement.

I like this video

Disagree  Agree

I share this video with friends

Not share  share

I agree with the feedback of other people about this video with the comments function

Disagree  Partially disagree  No opinion  Partially agree  Agree

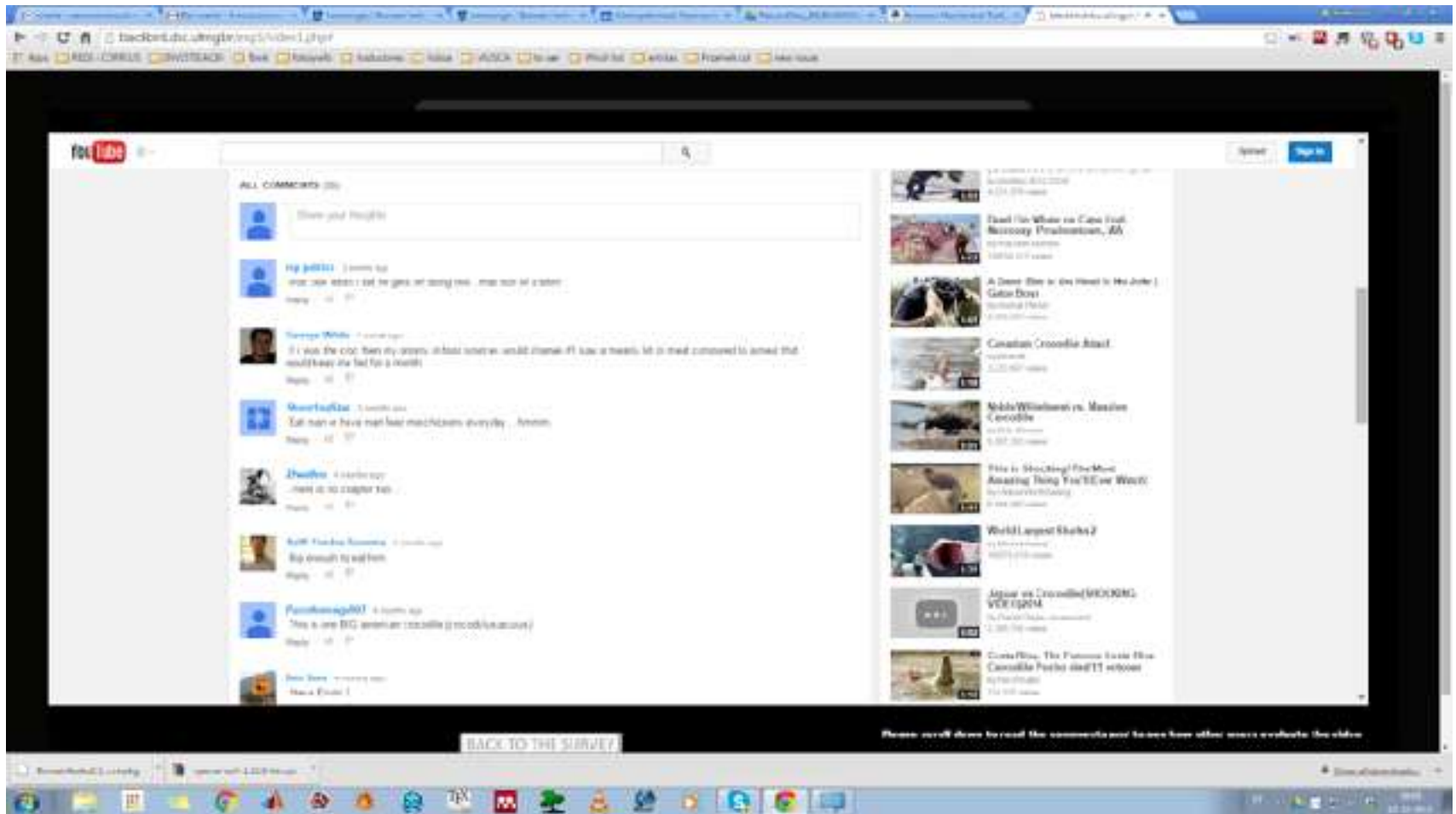
I'd like to see a comment or give a thumbs up/down to this video

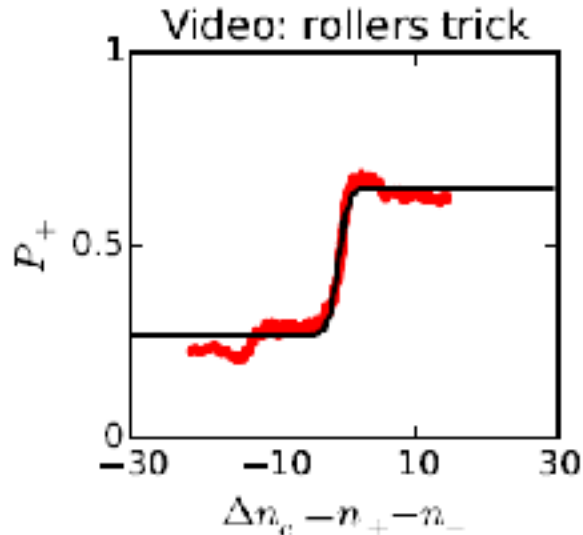
No  Yes

I have bookmarked this video

No  Yes

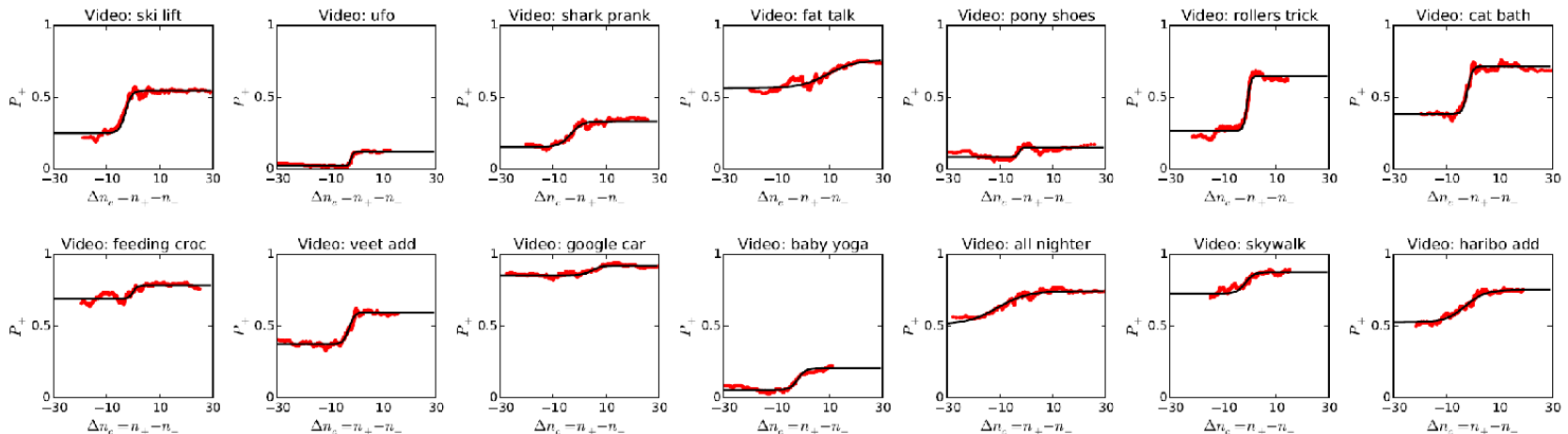
Next video





You are more likely to like/dislike a video if people liked/disliked it in the comments

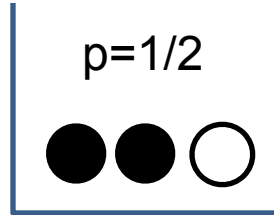
BUT some people are 'immune' to influence



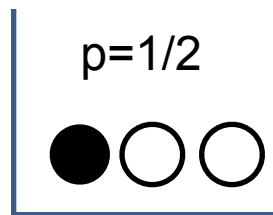
# And social interactions?

Two types of boxes

Majority black (MB)

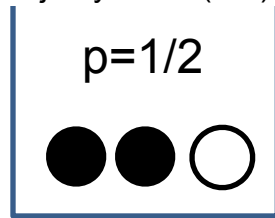


Majority white (MW)

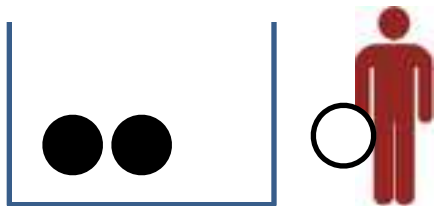
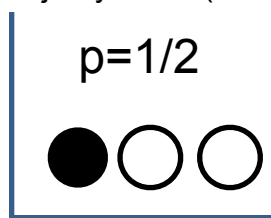


# And social interactions?

Majority black (MB)

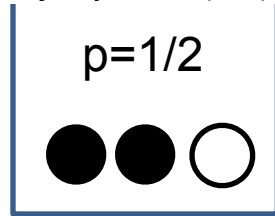


Majority white (MW)

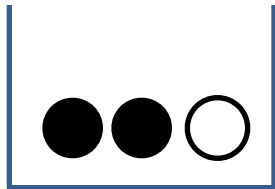
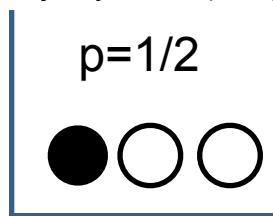


# And social interactions?

Majority black (MB)



Majority white (MW)

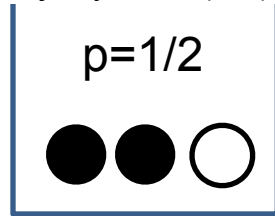


MW

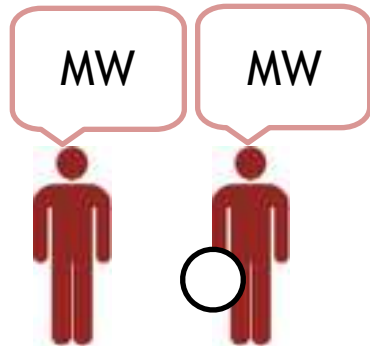
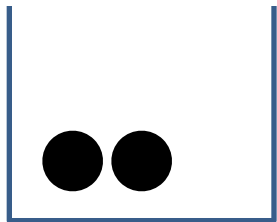
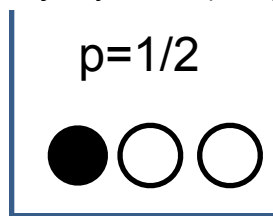


# And social interactions?

Majority black (MB)



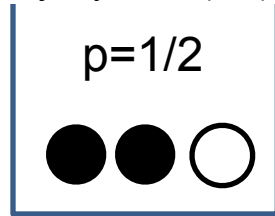
Majority white (MW)



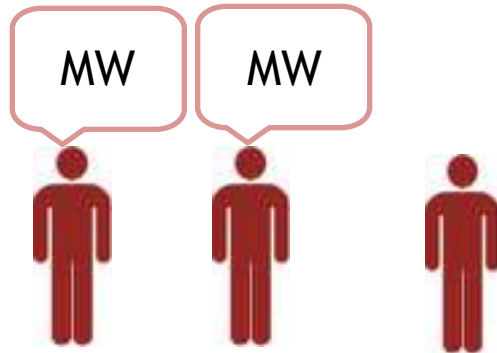
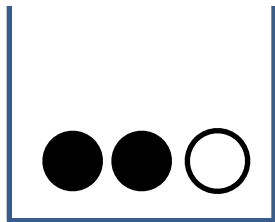
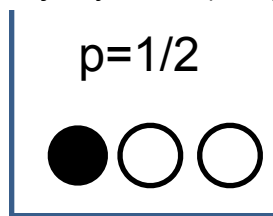


# And social interactions?

Majority black (MB)

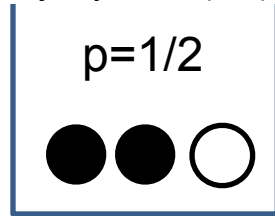


Majority white (MW)

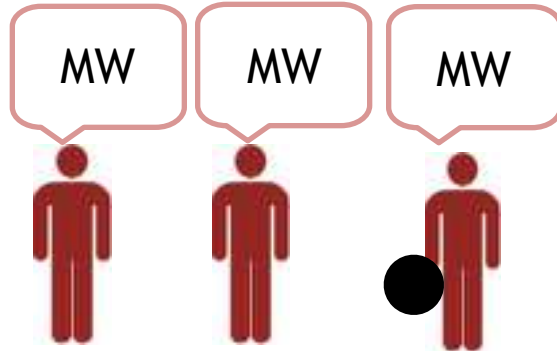
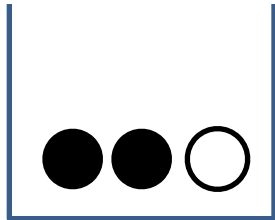
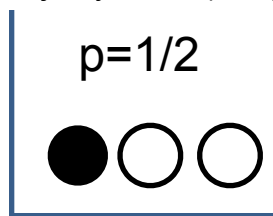


# And social interactions?

Majority black (MB)

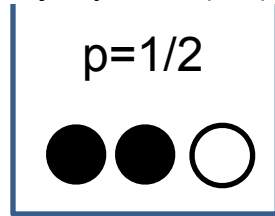


Majority white (MW)

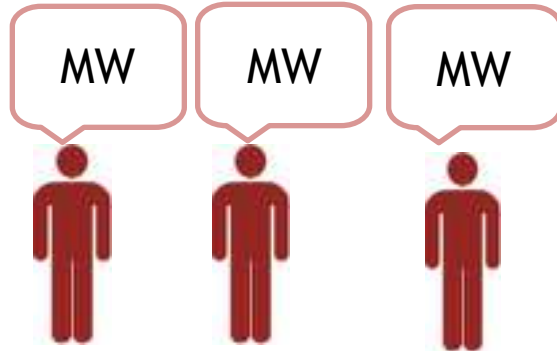
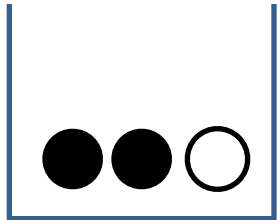
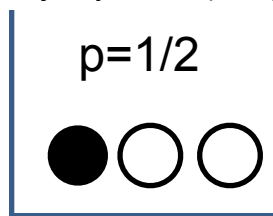


# And social interactions?

Majority black (MB)

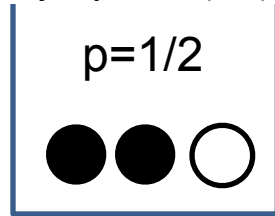


Majority white (MW)

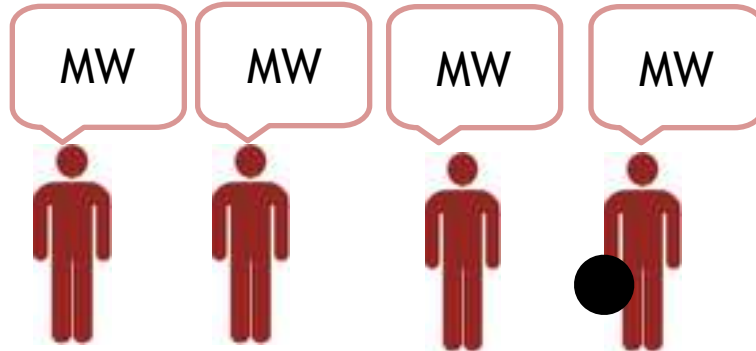
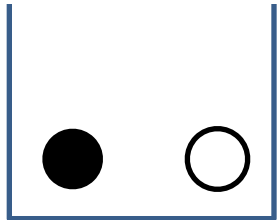
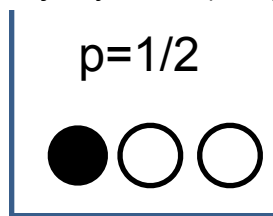


# And social interactions?

Majority black (MB)

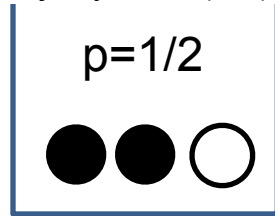


Majority white (MW)

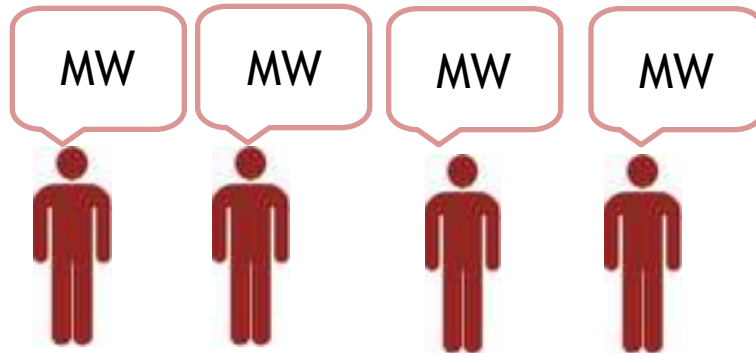
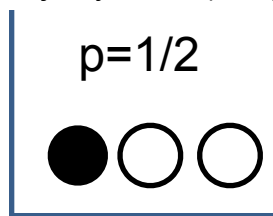


# And social interactions?

Majority black (MB)

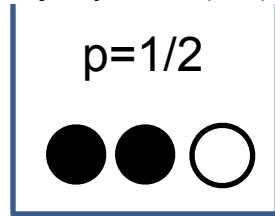


Majority white (MW)

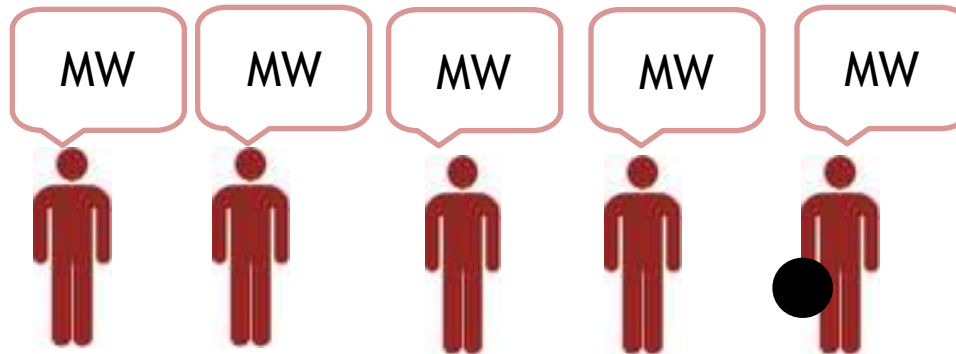
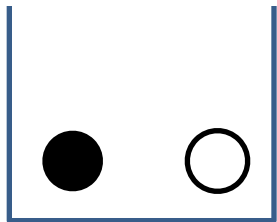
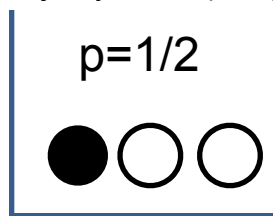


# And social interactions?

Majority black (MB)

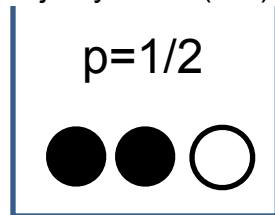


Majority white (MW)

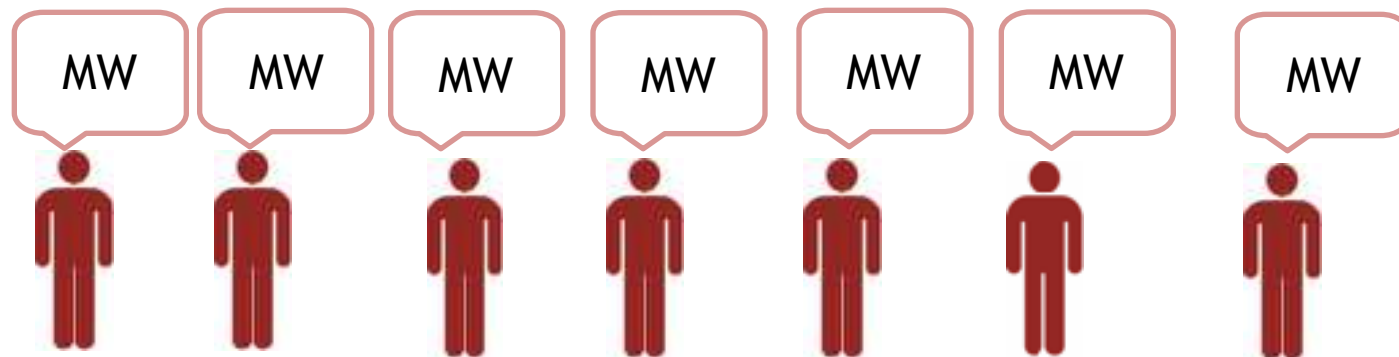
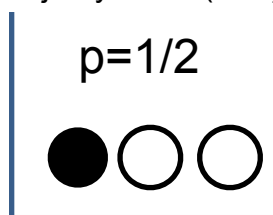


# And social interactions?

Majority black (MB)



Majority white (MW)



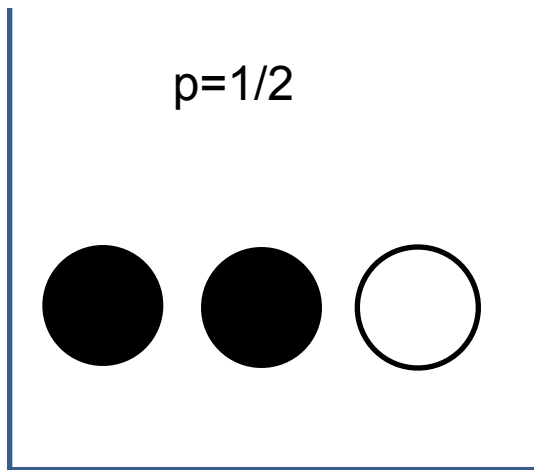
Social interactions can cascade an error EVEN if everyone is choosing the best they

Banerjee, Quart J Econ (1992)

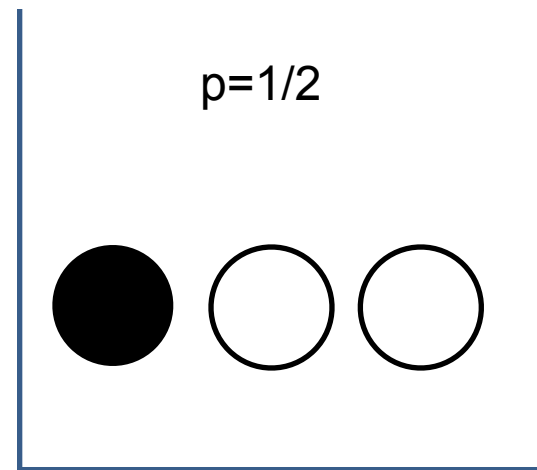
Bikhchandani, Hirshleifer, Welch (1992) J Polit Econ

## Information cascades

Assumes people are super-rational and decide in sequence



Majority black (MB)



Majority white (MW)

You draw one ball and with that ball and what people have estimated before (MB or MW), you make an estimation (MB or MW)



## Rational decision rule

Guess MW when  $P(\text{MW} | \text{what you see and heard}) > 1/2$   
Else choose MB

## From the setup we know

$$P(\text{MW}) = P(\text{MB}) = 1/2$$

$$P(w | \text{MW}) = P(b | \text{MB}) = 2/3$$

## Decision by first person assuming she draws w

$$P(\text{MW} | w) = P(w | \text{MW})P(\text{MW}) / P(w) = 2/3 \times 1/2 / 1/2 = 2/3 > 1/2 \rightarrow \text{says MW}$$

where

$$P(w) = P(w | \text{MW})P(\text{MW}) + P(w | \text{MB})P(\text{MB}) = 2/3 \times 1/2 + 1/3 \times 1/2$$

...

## Third person

$$P(MW | w w b) = P(w w b | MW) P(MW) / P(w w b) = 4/27 \times 1/2 / 1/9 = 2/3 > 1/2 \rightarrow \text{says MW}$$

where

$$P(w w b | MW) = P(w | MW) P(w | MW) P(b | MB) = 2/2 \times 2/3 \times 1/3 = 4/27$$

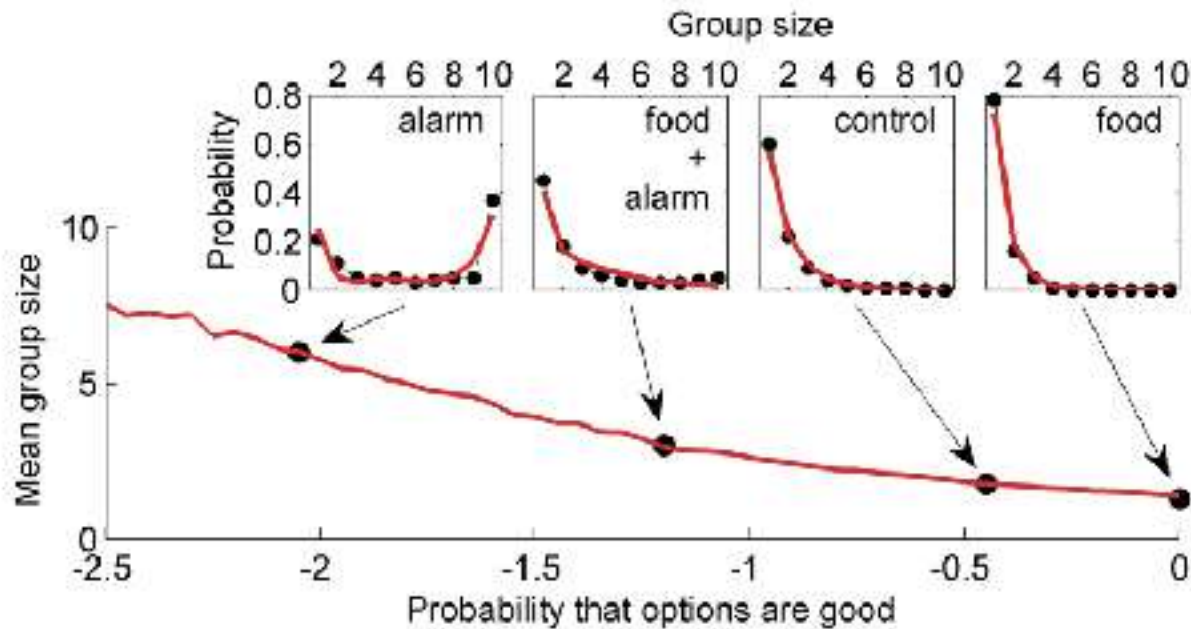
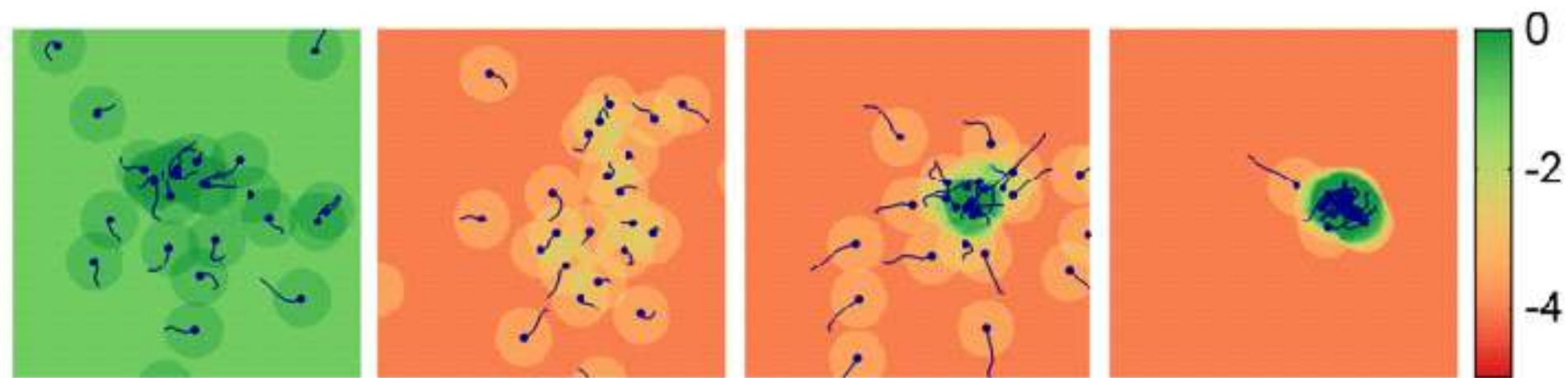
$$P(MW) = 1/2$$

$$P(w w b) = P(w w b | MW) P(MW) + P(w w b | MB) P(MB) = 4/27 \times 1/2 + 1/3 \times 1/3 \times 2/3 \times 1/2 = 1/9$$

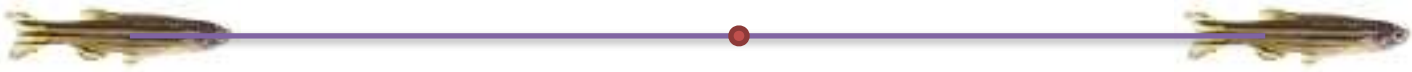
The third person should choose w independently of what she sees  
The same for the rest, so a cascade forms

It is rational to form a cascade in which people simply copy,  
and they might get it wrong

# Increased aggregation in adversity from decision-making



Perez-Escudero & de Polavieja (unpublished); data from Hoare et al (2004)



$$v_F(0)$$

$$d$$

$$v_L$$

$$v_F(0) > v_L$$

$$v_F(t_{eq}) = v_L$$

$$d_F(t_{eq}) = d + d_L(t_{eq})$$

$$v_F(t_{eq}) = v_L$$

$$\frac{dv_F}{dt} = -\alpha v_F$$

$$v_F(t) = v_F(0) \exp(-\alpha t)$$

$$t = \frac{1}{\alpha} \ln \frac{v_F(0)}{v_F(t)}$$

$$t_{eq} = \frac{1}{\alpha} \ln \frac{v_F(0)}{v_L}$$

$$d_F(t_{eq}) = d + d_L(t_{eq})$$

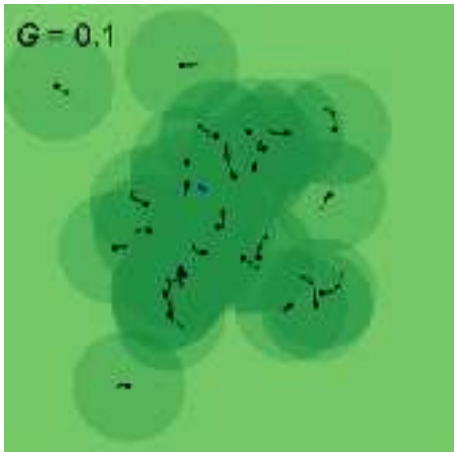
$$v_F(t) = v_F(0) \exp(-\alpha t)$$

$$d_F(t_{eq}) = \int_0^{t_{eq}} v_F(0) \exp(-\alpha t) dt = \frac{1}{\alpha} (v_F(0) - v_L)$$

$$d_L(t_{eq}) = v_L t_{eq} = \frac{v_l}{\alpha} \ln \frac{v_F(0)}{v_L}$$

$$\frac{1}{\alpha} (v_F(0) - v_L) = d + \frac{v_l}{\alpha} \ln \frac{v_F(0)}{v_L}$$

# Other uses of modelling in collectives



Approach as acceleration until gliding alone

takes a fish to another fish

Laan & de Polavieja (submitted)



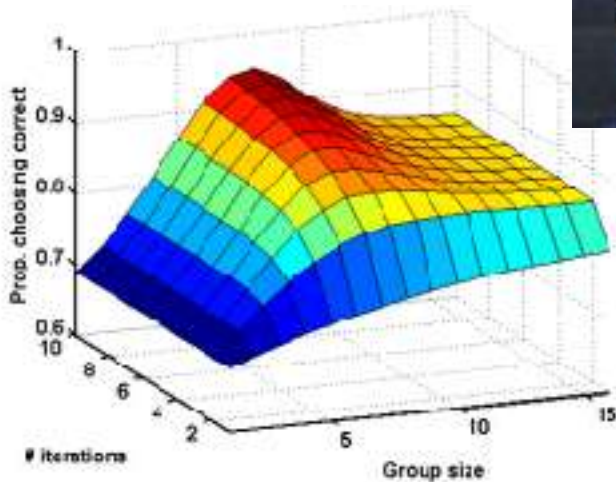
Consensus in kids using a geometric mean

Ioannou, Madirolas & de Polavieja (in prep)



Panic is decision-making

Perez-Escudero & de Polavieja (submitted)



Most accurate groups are small

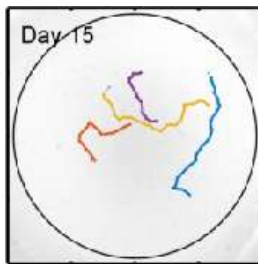
Vicente-Page & de Polavieja (submitted)

Subjective opinion on videos obeys Bayesian rules



$$P_x = \left( 1 + \frac{1 + aS^{-(n_x - kn_y)}}{1 + aS^{-(n_y - kn_x)}} \right)^{-1}$$

Theory of decision-making in groups



Collective behavior in zebrafish



Collective behavior in humans